LPR VI Cup

9.s06.e05

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

   where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative task

1 Kinematics of SRT

1.1. Two events occur in the laboratory frame of reference in the same place, but they are separated in time by 3 seconds.

(a) *(0.5 points)* What is the distance in space between these events in the rocket's reference frame, if the time interval between events is 5 seconds in it?

(b) *(0.5 points)* What is the velocity v_r of the rocket relative to the laboratory reference frame?

1.2. *(2 points)* A spaceship moves at a constant speed $V = (24/25)c$ toward the center of the Earth. What distance, in the reference frame associated with the Earth, will the spaceship travel during a time interval $\Delta t' = 7$ s as measured by the ship's clock? Neglect the rotation of the Earth and its orbital motion.

1.3. *(2 points)* A spaceship is flying at a speed of $V = 0.6\,c$ from one space beacon to another. At the moment when it is exactly halfway between the beacons, each beacon emits a light pulse toward the ship. Find the time interval, as measured on the ship, between the moments when these pulses are detected. The distance between the beacons is such that light takes 2 months to travel from one to the other.

1.4. *(2 points)* Two starships with their engines off are moving toward each other. On one starship, signal lights flash simultaneously at the bow and stern every second. On the approaching starship, every 0.5 seconds, two flashes are observed with a time interval of $\tau' = 1$ μs. Find the length l_0 of the first starship and their relative approach speed v.

2 Classical black hole

During prolonged observation of the position of a star near the center of the galaxy, it was found that the star undergoes periodic motion along a circular path in the gravitational field of a certain massive object. It is known that the distance from the observation point to the center of the galaxy is $\approx 26 \cdot 10^3$ light-years, the period of the star's orbit is ≈ 16 years, the radius of the star's circular trajectory as seen from the observation point is visible at an angle of $88 \cdot 10^{-3}$ arcseconds, and the plane in which the star moves is perpendicular to the direction of observation.

2.1. *(1.5 points)* Determine the mass of the object attracting the star.

2.2. *(1.5 points)* Assuming the object attracting the star is spherically symmetric and its size is sufficiently small, determine the boundaries of the region around such an object from which no signal can reach a distant observer.