LPR VI Cup

9.s06.e02

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative task

The door to the workshop swung open. With a relatively significant delay—one for which there was no one to scold him—a ringing stream of apologies tumbled into the hallway first, followed by the owner of the voice: Hans's loyal apprentice, Martin.

Shaking off the water and turning out the leather jacket thrown over his head, he stopped mid-sentence in his excuses and froze, wearing the not-uncommon stunned expression for someone in this place. A lot had changed since his last visit.

Where there had once been familiar order and determinism, now there was mess and chaos. Martin began carefully and gently sorting through the scattered things, trying to somehow organize them, when his gaze fell on a small note sticking out of the drawer assigned to him.

"Martin, like all of us, I have very little time. But I think I've found something that could solve this problem. I'll be gone for a little while, but we'll definitely meet again in the future. While I'm away, please look after Snoopy and yes, I haven't forgotten about the clock assignments—I'll check them as soon as I return. P.S. I'd appreciate it if you could help tidy up the workshop. As usual, I haven't managed to do anything, and that's unlikely to change anytime soon — Hans"

Sighing with some relief, Martin set about cleaning up, so that afterwards he could return to the study of clock mechanisms that his older friend had entrusted to him.

Part I

It is known that due to the friction force in the hourglass, the speed of sand pouring from one container to another does not depend on the height of the sand column. Using the dimension method, find up to a constant value:

1. *(2 points)* The dependence of the mass flow rate in such an hourglass on the diameter d of the neck of the tube connecting the two containers.

Assume the density of the sand is ρ_0, and the acceleration due to gravity is known and equal to g. Assume that the friction coefficient is included in the constant factor.

Part II

The length of a simple pendulum was increased by 2%.

1. *(0.5 points)* Determine by what percentage its period will increase.

A clock with a simple pendulum is placed in an experimental platform moving vertically downward with a constant acceleration $a < g$. Find:

1. *(0.5 points)* The ratio of the periods of the simple pendulum in a stationary and in the accelerating experimental platform.
2. *(0.5 points)* The daily error of such a clock.

Two stationary walls are at a distance L from each other. A small body is at rest near the left wall on a smooth horizontal table. The body is given an initial velocity v_0 directed toward the right wall. It is known that after each collision with a wall, the speed of the body decreases by a factor of 1.2.

1. *(0.5 points)* After what time τ will the body hit a wall for the tenth time?

Part III

Let us consider a spiral spring, coiled in the shape of a snail so that one end is attached to a shaft and the other to a winding drum. The figure on the left shows the spring in the fully unwound state, and on the right in the fully wound state. In the first case, the number of coils is n_1, in the second case the number of coils is n_2. If the spring is completely removed from the winding drum, then in its free (unstressed) state it will have n_0 coils.

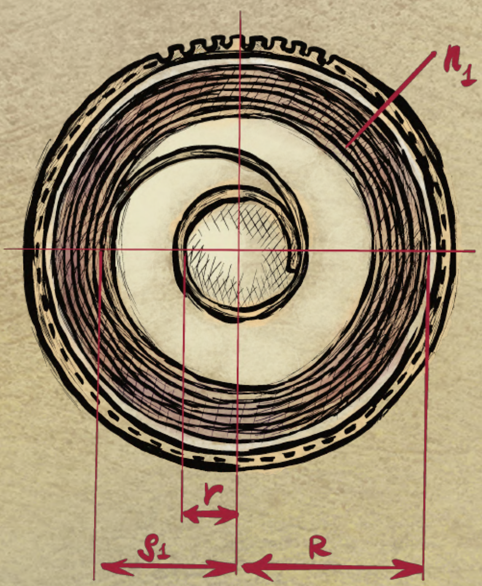

Figure 1: Position of the spring in the drum in the unwound state

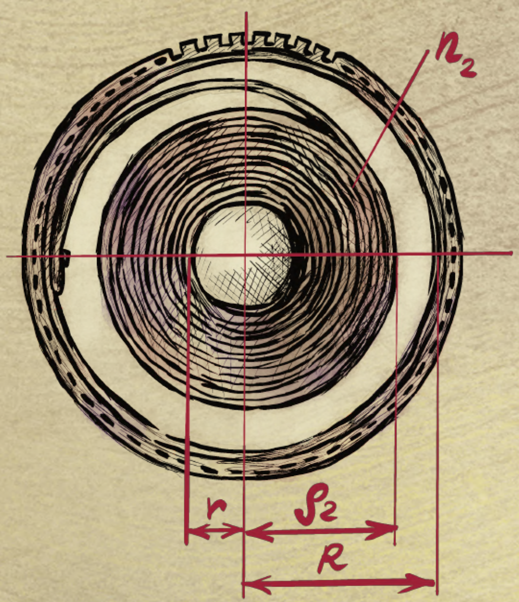

Figure 2: Position of the spring in the drum in the wound state

1. *(0.5 points)* Determine the range in which the number of coils of the spring fixed in the drum varies.
2. *(0.5 points)* Determine at which number of coils in the spring the minimum and maximum torque occur.
3. *(1 point)* Draw a qualitative graph of the dependence of the torque M on the number of coils n and indicate on it all characteristic states of the spring, if the state corresponding to the spring removed from the system is $n = 0$.

Let's consider a weight-driven engine. It consists of a weight, a chain, and a sprocket (see figure). The chain (2), looped over the drum (3), is attached at one end to the weight (1). The other end of the chain remains free during operation. To prevent the chain from slipping off the drum, a hook is attached to its free end, which catches on a special stopper as the chain unwinds. Let R_s be the mean radius of the sprocket, N its rotational frequency, and D the diameter of the drum.

1. *(1 point)* Determine the time t it takes for the weight to rise to a height H.
2. *(1 point)* What is the torque on the axis of the drum?

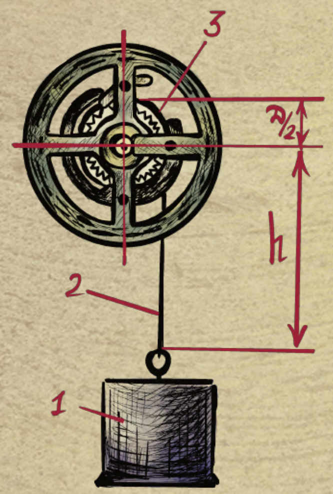

Figure 3: Diagram of the weight-driven engine. 1 – weight, 2 – chain, 3 – sprocket.

Part IV. The Anchor

In this section, you are again invited to consider the mechanics of the clock's ticking process.

We will analyze the motion in two planes: the plane of the balance wheel and the plane of the anchor, gear train, and mainspring. These planes are connected to each other via a stop in the form of an impulse jewel, which crosses both planes and is rigidly fixed to the balance wheel disk at some distance from its axis of rotation (see Fig. 2).

The balance wheel generates a periodic signal, the frequency of which we will later use to set the time scale step.

The anchor, gear train, and mainspring are used so that, at the moment the balance wheel passes through its equilibrium position, the anchor comes into contact with it via the impulse jewel for a short period of time, thereby "replenishing" its energy without significantly changing the period.

In more detail, the working cycle of the mechanism proceeds as follows:

Unlocking phase:

- The impulse jewel moves counterclockwise and enters the fork horn of the anchor.
- The first collision of the impulse jewel with the inner surface of the anchor fork horn occurs.
- The rotation of the anchor releases the gear wheel.

Impulse phase:

- The released gear wheel begins to rotate under the torque of the wound mainspring.
- Almost simultaneously, the second and third collisions occur in the anchor system, accompanying the start of the balance wheel's acceleration. The anchor fork transmits torque to the balance wheel via the impulse jewel, resulting in an impulsive acceleration of the oscillating part.

Drop phase:

- The impulse jewel exits the anchor fork.
- Almost simultaneously, the fourth and fifth collisions occur, accompanying the locking of the gear wheel by the anchor through contact with its next tooth and the braking of the anchor against a limiting stop.

After this, the oscillating mechanism, having been released and received a portion of energy from the mainspring, continues its harmonic motion, reaching the position of maximum deviation, after which it reverses direction and **begins to move in the opposite direction, repeating all phases of the clock mechanism cycle. Assume that during the reverse motion, the acceleration process in the impulse phase is absent due to the negligible interaction between the reverse side of the anchor and the balance wheel.**

1. *(2 points)* Assuming the torque of the mainspring is constant and equal to M, the effective torque transmission coefficient to the balance wheel through the gear wheel, gear train, anchor, and impulse jewel between impacts 3 and 4 is n, the total moment of inertia of the balance wheel is I and is much greater than the moments of inertia of the gear wheel, gear train, and anchor, the torsion modulus of the balance spring is k, the angular amplitude of oscillations is φ, and the time between the third and fourth impacts is t, determine what fraction of the energy stored in the oscillatory motion is lost by the balance wheel in one period of oscillation.

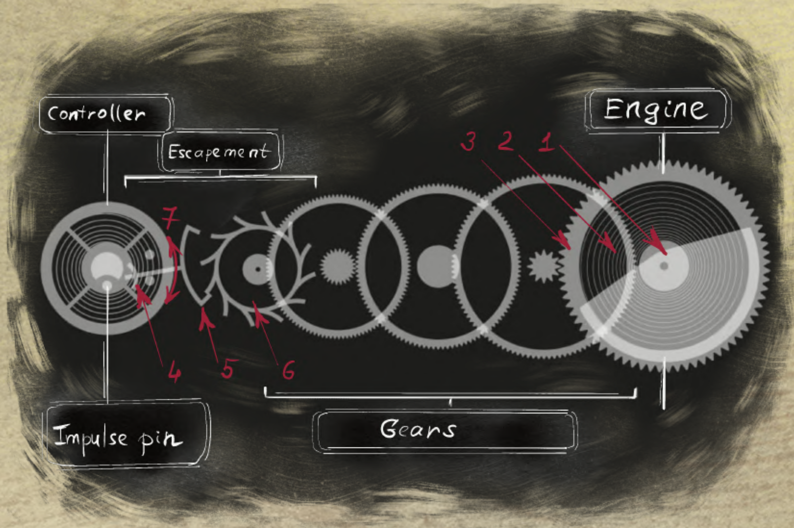

Figure 4: Diagram of a clock mechanism. 1 – shaft, 2 – mainspring, 3 – barrel drum, 4 – fork horn, 5 – pallet, 6 – escape wheel, 7 – direction of fork movement. The mechanical movement takes place in two parallel planes, which are themselves parallel to the plane of the drawing. The controller is located in the first plane, while the escapement, gears, and engine are located in the second plane. Interaction between the planes occurs only through the impulse pin.