LPR Cup

9.s05.e04

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

   where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Special case

Time flew by unnoticed, and when the hands of the clock had covered a significant portion of the circle, Hedgehog finally stretched his stiff needles and decided to stroll around the observatory room. A bookcase caught his eye, and he discovered several intriguing books and notes within. He snorted with delight and began to examine them intently.

Theoretical part

Classical Dimensional Analysis

Example 1. Galileo's Problem Determine, up to a constant factor, the distance traveled in a given time by a body freely falling from rest.

Solution

Step 1. Identifying the determining parameters. In this example, the distance s depends on the time of motion t and the acceleration due to gravity g. It is also reasonable to assume that the distance may depend on the mass of the body m.

Step 2. Setting up and solving a system of equations. We write the equation in the form:

$$s = f(g,t,m),$$

where $f(g,t,m)$ is a power function of its variables of the form $f = \mathrm{Const} \cdot g^\alpha t^\beta m^\gamma$. This gives us an equation for the dimensions:

$$L = L^\alpha T^{\beta-2\alpha} M^\gamma,$$

where L, T, M are the units of length, time and mass, respectively. From this, we obtain a system of equations:

$$\begin{cases} 1 = & \alpha \\ 0 = & \beta - 2\alpha \\ 0 = & \gamma. \end{cases}$$

Solving this system of equations, we find that $\alpha = 1$, $\beta = 2$, $\gamma = 0$. Thus we find the answer:

$$s = \mathrm{Const} \cdot gt^2.$$

Note that we also found that the distance does not depend on the mass of the body. When using dimensional analysis, the terminology of dimensionless parameters is also used. In the previous example, one could say that we have one dimensionless parameter $\pi_1 = s/(gt^2)$. **Example 2. Tension in a ring.** Find, up to a constant factor, the tension force in a wire ring rotating in its own plane about an axis perpendicular to the plane of rotation and passing through the center of the ring. **Solution Step 1. Identifying the determining parameters.** We are interested in the dependence of the force F on the angular velocity ω. Since the problem is directly related to dynamics, the result may depend on the mass of the ring m, as well as on the geometry of the problem, which is uniquely determined by the radius of the ring R. **Step 2. Setting up and solving a system of equations.** Using the dimensional analysis method, we obtain the equation:

$$F = \mathrm{Const} \cdot m^\alpha R^\beta \omega^\gamma.$$

Since force has dimensions of LMT^{-2}, we obtain an equation for the powers:

$$LMT^{-2} = M^{\alpha}L^{\beta}T^{\gamma}.$$

From which we get $\alpha = 1$, $\beta = 1$, $\gamma = -2$, and the force is determined by the expression:

$$F = \text{Const} \cdot mR\omega^2$$

Note that in this problem, only one dimensionless parameter can be formed $\pi_1 = \dfrac{F}{mR\omega^2}$. **Example 3. Nicholson's Hydrometer.** Find, up to a constant factor, a formula for the period of oscillation of a cylinder of mass m floating in a liquid when it is slightly displaced downward from its equilibrium position. **Solution Step 1. Identifying the determining parameters.** In this problem, this step is not as straightforward as in previous examples. It is clear that the period t may depend on the mass of the hydrometer m, must be determined by the buoyant force, which depends on the density of the liquid ρ and the acceleration due to gravity g, and must also depend on the geometry of the problem, which is given by the radius of the base of the cylinder or, equivalently, its area S. Note that the geometry of the problem does not depend on the height of the cylinder, since when it changes, the depth of immersion changes proportionally. **Step 2. Setting up and solving a system of equations.** From the previous point we get the formula:

$$t = \text{Const} \cdot m^{\alpha}s^{\beta}\rho^{\gamma}g^{\delta},$$

which is equivalent to:

$$T = M^{\alpha}L^{2\beta}L^{-3\gamma}M^{\gamma}L^{\delta}T^{-2\delta}$$

From where we get the system of equations:

$$\begin{cases} 0 = & 2\beta - 3\gamma + \delta \\ 0 = & \alpha + \gamma \\ 1 = & -2\delta. \end{cases}$$

We have obtained a system of three equations and four unknowns. One of the variables will remain unknown and can be found from experiment (or by applying the extended dimensional analysis method, which you will be asked to do in the Alternative Problem). Leaving as a parameter through which other quantities are expressed, the power α we get that $\beta = -\dfrac{3\alpha}{2} + \dfrac{1}{4}$, $\gamma = -\alpha$, $\delta = -\dfrac{1}{2}$. I.e.

$$t = \text{Const} \cdot m^{\alpha}s^{-3\alpha/2+1/4}\rho^{-\alpha}g^{-1/2}.$$

In this example, two dimensionless parameters can be composed: $gt^2/\sqrt{s}$ and $ms^{3/2}/\rho$.

Extended Dimensional Analysis

Example 4. Extended Dimensional Analysis. A bullet is fired with an initial velocity u in the horizontal direction at a height h from the ground. Determine the horizontal range of the bullet. **Solution** The determining parameters of this problem are the horizontal range L, the initial velocity of the bullet u, the acceleration due to gravity g, and the height at the moment of the shot h. We obtain the formula:

$$L = \text{Const} \cdot u^{\alpha}h^{\beta}g^{\gamma},$$

from which we obtain the formula for the dimensions:

$$L_x = \left(L_x T^{-1}\right)^{\alpha} L_z^{\beta} \left(L_z T^{-2}\right)^{\gamma}.$$

Solving this system of equations, we find that $\alpha = 1$, $\beta = 1/2$, $\gamma = -1/2$, whence we find that the range is equal to:

$$L = \text{Const} \cdot v\sqrt{\frac{h}{g}}.$$

Note. Using the classical method of dimensions, one unknown power will remain.

Alternative Problem

1. *(1 point)* Using the method of dimensions, find the period of oscillation of a simple pendulum and show that it does not depend on the mass of the weight.
2. *(2 points)* A bullet is fired with an initial velocity u at an angle α to the horizontal plane. Using the extended method of dimensions, find the range up to a constant factor.
3. *(3 points)* Using the extended dimensional analysis, find, up to a constant factor, the period of oscillation of Nicholson's hydrometer.
4. *(4 points)* The table shows the current-voltage characteristic of a vacuum tube. It presents the readings of the anode current I versus the voltage U (see table). Assuming that $I = \alpha U^{\beta}$, determine α and β.

№	U, V	I, μA
1	0,5	3,24
2	1,0	12,7
3	1,5	24,62
4	2,0	38,05
5	2,5	55,54
6	3,0	73,2
7	4,0	116,82
8	5,0	158,14
9	12,0	188,82
10	15,0	911,9
11	20,0	1320,5
12	60,0	1634