9.s05.e01

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

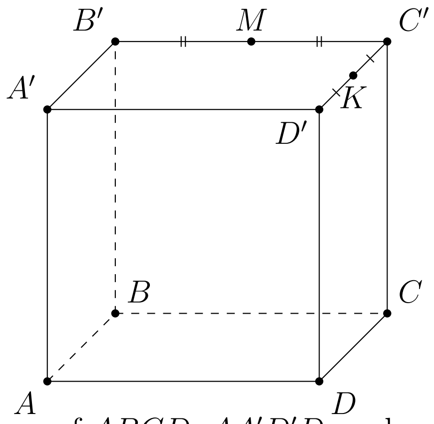
$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative task

Part 1. Ant and Cube



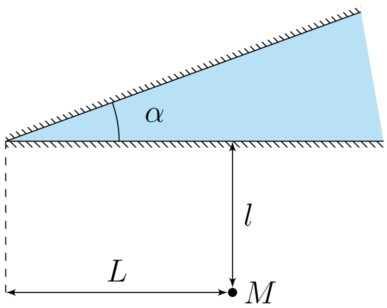
Traveler-Ant met LPR Cube in Cuba and, out of professional habit, decided to move between its pre-selected points in minimal time. In each of the points, Ant starts from the point A. The face length of LPR Cube is a. In all items, out of respect for Cube, the Ant can only move along the faces $ABCD$, $AA'D'D$, $A'B'C'D'$, $BCC'B'$.

1. *(0,5 points)* Ant moves along all the faces available to him with the same velocity v. What is the minimum time t_{AM} Ant will be able to get from the point A to the point M — the middle of the edge $B'C'$?

2. *(2 points)* Ant moves with a constant velocity v along the faces of $ABCD$, $AA'D'D$, and along the faces of $A'B'C'D'$, $BCC'B'$ with a constant velocity $1{,}3v$. What is the minimum time t_{AK} Ant will be able to get from the point A to the point K, the middle of the edge $D'C'$?

3. *(0,5 points)* What is the minimum time t_{AD} he will be able to run with a constant velocity v to the point D, if along the way he wants to run to the edge $A'D'$?

Part 2. How not to get into a trouble puddle



Along the journey Ant ended up next to a puddle and wanted to cross it. On land, Ant runs with a speed v, and his she-friend, a Water Strider, can drive him through a puddle with a speed $v/\sqrt{2}$.

The angle α is equal $30°$. The geometric parameters are shown in the figure. It is known that $L = 2l$. At the initial moment of time, Traveler-Ant is located at the point M.

1. *(0,5 points)* In what time will he reach the opposite «shore» of the puddle if he always moves perpendicular to the next intersected boundary?

2. *(0,5 points)* In what time will he reach the opposite «shore», if his velocity is always directed at an angle $60°$ to the nearest edge of the puddle, so as to approach the vertex of the angle while moving?

3. *(0,5 points)* In what time will he reach the opposite «shore», if his velocity is always directed at an angle $30°$ to the nearest edge of the puddle, so as to approach the vertex of the angle while moving?

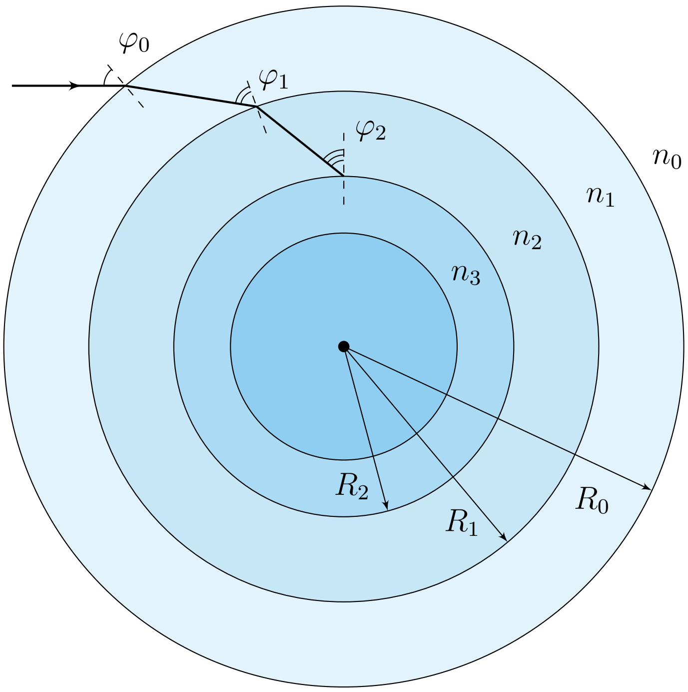
4. *(3 points)* What is the minimum possible time it takes for Ant to move the opposite «shore»?

Assume that the distance l is known. In paragraphs 2.3, the angle is specified relative to the horizontal border.

Part 3. Sphere

A ray of light falls at an angle $\varphi_0 = 45°$ on an optical system consisting of concentric spheres of different radii and different refractive indeces. The radii of the spheres are $R_N = R/2^N$, and the refractive indices are $n_N = n_0 \cdot 2.5^N$, where N is the number of the sphere (see figure). Find:

1. *(2,5 points)* the angle of incidence of the ray φ_{34} when passing through the interface between media 3 and 4?



Mathematics software may be useful for you to solve some of the items. Numerical answers must be presented with an accuracy of at least 1%.