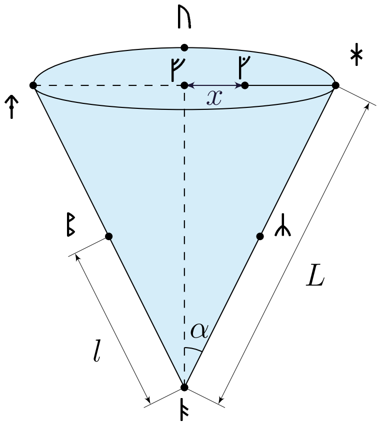
LPR Cup

9.s05.e01

Do you know what your problem is, Antz?
You're thinking too much.
Antz

Pilgrim Ant

Traveler – Ant can crawl along the lateral surface and base of a newspaper bag in the form of a right circular cone with an angle of α. The slant length of the bag is L. In the paragraphs 1 – 4 Ant starts from the letter ᛒ ~~with the words «Let's ᛒegin!»~~ printed on external surface of the bag at a distance of l from the apex of the cone.



In the paragraphs 1 and 2, the bag is closed and filled with seeds, and Ant can travel both along the side surface and along the base of the cone. In the paragraphs 3 and 4, the base of the bag is carefully torn off and Ant can only run along its side surface.

Find:

1. *(1 point)* The minimal time $\tau_{\text{ᚼ}}$ it takes Ant to reach the letter ᚼ by moving at a constant speed v; the letter ᚼ is located on the border of the base of the bag in the same plane with its axis ᚭᚠ and the letter ᛒ (see figure); In this paragaph $\alpha = \pi/6$.

2. *(1 point)* The minimal time $\tau_{\text{ᚵ}}$ it takes Ant to reach the letter ᚵ, moving at a constant speed v; the letter ᚵ is printed on the segment ᚠᚼ and is located at a distance x from the letter ᚠ (see figure). To simplify the analysis, when calculating in this paragraph, take $l = 0{,}5L$, $\alpha = 0{,}8$ rad, $x = 0{,}7R$, where R is the radius of the circle at the base of the bag;

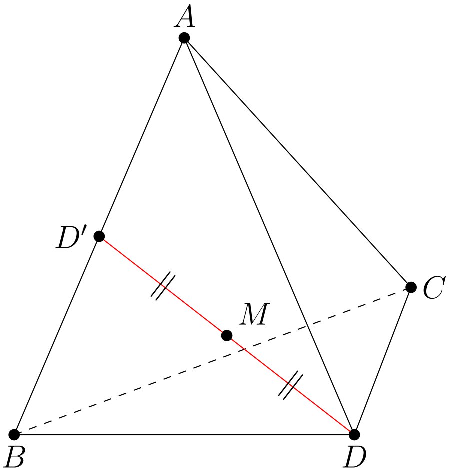
3. *(1 point)* The minimal time $\tau_{\text{ᛦ}}$ it takes Ant to reach the letter ᛦ, which is symmetrical to the letter ᛒ relative to the axis ᚭᚠ (see figure). At the same time, Ant wants to see what is printed on the inner surface of the bag, so it first runs to its base at a speed of v, quickly looks inside, after which it crawls to the letter ᛦ, but under the weight of the acquired knowledge, its speed becomes $v/2$. At this point, the bag is folded so that $\alpha = \pi/4$, and $l = 0{,}6L$.

4. *(1 point)* The minimum time $\tau_{g\text{ᚼ}}$ it takes Ant to reach the letter ᚼ, provided that it first moves along the side surface of the bag to any arbitrary point forming ᚭᚢ at a constant speed v, then from this point on ᚭᚢ to the letter ᚼ at a constant speed $v/2$ (see the same picture); point ᚢ is located on a circle at the base of the bag in the middle of the arc ᛏᚢᚼ; at this point, the bag is folded so that $l = L/2$, $\alpha = \pi/6$;

5. *(3 points)* Ant got carried away reading the newspaper from which the bag was made and noticed an interesting letter ℌ at a distance of $l/2$ from the letter ᚭ. He began to move

towards it in such a way that his speed began to change according to the law $v(r) = a/r$, where a is an unknown constant, and r is the distance to the letter ᚨ. Ant really wanted to reach it as soon as possible, so he chose such a trajectory to get to the letter $\mathfrak{H}$ in the shortest possible time without losing sight of it, i. e. without making a single complete turn around the bag. Find the angle between the velocity vector of Ant at the beginning of the path and ᚨᛏ, if as it approached the letter $\mathfrak{H}$ it moved parallel to the base of the cone.

6. *(3 points)* Ant was blown away by a sharp gust of wind from the bag and when the wind died down, it found itself on a milk carton in the form of a regular tetrahedron. When it regained his sences, he found that he was sitting on the letter *M*, which turned out to be the middle of the height of *DD'*. To get a better look at the setting sun, Ant decided to run to the edge of AC and, out of professional habit, he wanted to do it in the minimal time. Ant crawls along the face of ABD at a speed of v, and on the faces of ACD and ACB live his old friends Caterpillar-Surveyor and Haymaker-Spider respectively, who are always ready to give a ride to the Traveler Ant on their face. The speed of Caterpillar is $\sqrt{3}v$, and the speed of Spider is $10{,}2v$. Since the milk carton is on the ground, Ant cannot move along the face of *BCD*. What is the minimum time it takes to get from the letter M to the edge AC? The length of the edge of the tetrahedron is a.



Mathematics software may be useful for you to solve some of the points. Numerical answers must be presented with an accuracy of at least 1%.

First hint — 29.04.2024 20:00 (Moscow time)
Second hint — 01.05.2024 12:00 (Moscow time)

Final of the first round — 03.05.2024 20:00 (Moscow time)