Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

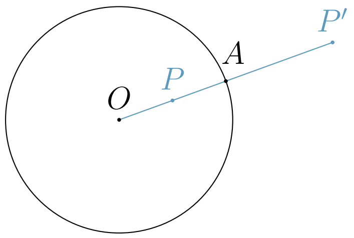
where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Alternative problem

Task 1. *(0 points)* Inversion is a transformation of the Euclidean plane. Let us define a point O (we call it the pole of inversion) and some distance R (we call it the radius of inversion). Then the inversion given by the parameters above translates some point P (not coinciding with O) to point P' in such a way that

1. P' lies on the line OP.
2. $OP \cdot OP' = R^2$

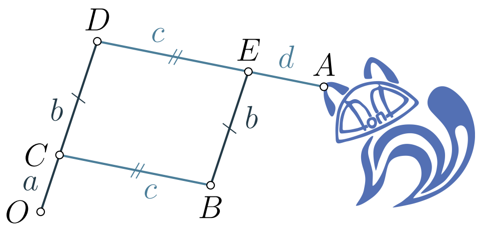


1. (*0 points*) Prove that by inversion, a circle with radius a centered in B (BO = b $\neq$ 0) turns into a circle.
2. (*0 points*) Find the radius and position of the center of the resulting circle.

Task 2. *(2 points)* As in the workplace of any detached creator or just a keen inventor, there was what an ordinary passerby would call a mess in Hans' workshop. Of course, Hans could have argued with anyone, who would have told him this personally, but at some point the idea of cleaning settled in his head. It was the pantry that took the blow the first, from which Hans, with a warm feeling of nostalgia, extracted one of his first professional instruments - a pantograph and an ellipsograph.

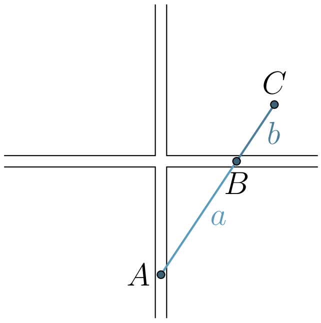
The pantograph is a construction of pivotally fixed inextensible rods OD, DA, BE, BC (see Fig.). The point O is always stationary. It is known that $OC = a$, $CD = BE = b$, $BC = ED = c$, $AE = d$,, and the ratio is: $b/a = d/c = \gamma$. All four rods can rotate relative to each other at the points of articulation – E, B, C, D. It is known that the point A drew a cat.

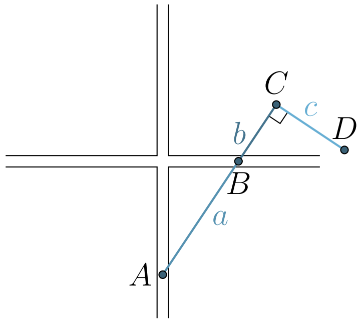
1. *(1 point)* Prove that the point B also drew a cat.
2. *(1 point)* How many times does the length of the tail of a cat drawn with the dot B differ from the length of the tail drawn with the dot N?



Task 3. *(2 points)* The ellipsograph is a mechanism consisting of two sliders - A and B, held together by rigid rod structures. The sliders can move along two infinite perpendicular guides. It is known that $AB = a$, $BC = b$, $CD = c$.

Find the trajectory of the point C in the first case and the point D in the second (see Figures), if it is known that the rod structures have made one complete rotation.



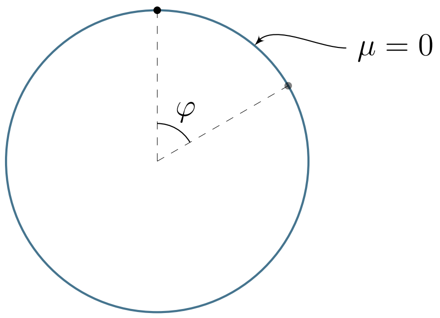


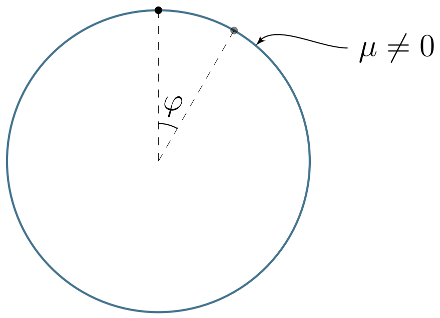
Task 4. *(0 points)* Digging into his old designs, Hans discovered a strange design, the reason

for the creation of which he had long forgotten. It consisted of a large vertically positioned hoop of radius R and a small ring put on it. Find all equilibrium positions of the ring on the hoop and examine them for stability if:

1. *(0 points)* The hoop is smooth.
2. *(0 points)* The coefficient of friction of the ring on the hoop is known and is equal to μ.

It is convenient to set the equilibrium position through the angle from the center of the hoop between the directions to the ring and the upper point of the hoop (see Fig.).



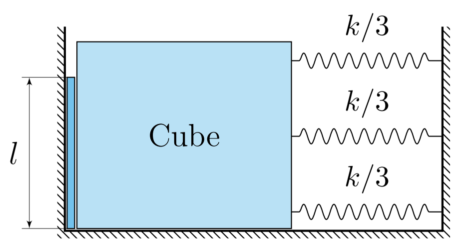


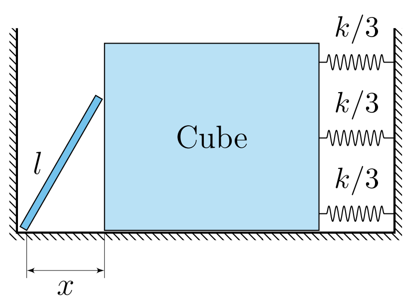
Task 5. *(4 points)* Attempts to recall the purpose of the design had to be interrupted, because Hans received an order for the manufacture of a non-standard organizer for vinyl records. It was non-standard because it contained a mechanism that was supposed to keep a horizontal stack of records from falling, pressing it against one of the walls of the organizer.

And so, a few hours later, the order was collected. It was a weightless cube on three springs of stiffness $k/3$ attached to the right wall of the organizer so that they were undeformed when the cube was at the opposite wall (see Fig.). The length of the undeformed springs was l_0, and all surfaces of the components of the organizer were smooth.

To test the mechanism, Hans put one plate from his collection into the product and noticed that it can be in equilibrium when the cube is at a distance of x from the left wall of the organizer (see Fig.). The Hans' plate is in a flat smooth square package with the side l and its center of mass coincides with the geometric center of the package. The weight of the plate together with the package is equal to m.

1. *(2 points)* At what x will the plate be in the equilibrium position?
2. *(2 points)* Are these equilibrium positions stable?

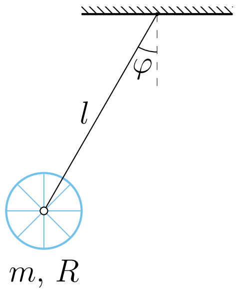




Task 6. *(2 points)* Having dealt with the organizer for the records, Hans decided to return to the previously found hoop and ring design. Realizing with annoyance that he didn't have a single chance to remember at least something about the purpose of the product, Hans took the ring off the hoop. Then he attached a number of weightless spokes to the hoop so that

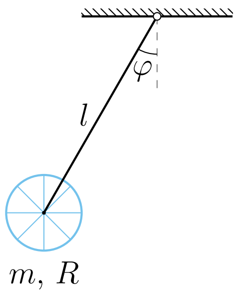
they symmetrically diverged from the center of the hoop to its surface. Hans hung the resulting “wheel” by the center to the ceiling on a weightless thread of length l, deflected it from the vertical by a small angle and released it (see Fig.). The radius of the hoop is R, the mass is m, the plane of the hoop is located vertically during movement. At some point, the angle between the thread and the vertical became equal to φ, and the rate of change of this angle was equal to $\dot{\varphi}$

1. *(0 points)* Write down the expression for the total energy of the system at this point in time.
2. *(0 points)* Derive from this expression by taking a derivative the tangential acceleration of the center of the hoop. Express the answers in terms of $l, R, m, g, \varphi, \dot{\varphi}$.



If someone had watched how closely and enthusiastically Hans followed the movement of the suspended hoop, he would have realized for sure that the inventor was not on the point of stopping. Quickly finding everything he needed in the workshop, Hans replaced the thread with a solid weightless rod, which was rigidly attached to the center of the “wheel” with one end, and pivotally to the ceiling with the other. The radius of the hoop is R, the mass is m, the plane of the hoop is located vertically during movement. At some point, the angle between the rod and the vertical became equal to φ, and the rate of change of this angle was equal to $\dot{\varphi}$

3. *(0 points)* Write down the expression for the total energy of the system at this point in time.
4. *(0 points)* Derive from this expression by taking a derivative the tangential acceleration of the center of the hoop. Express the answers in terms of $l, R, m, g, \varphi, \dot{\varphi}$.



Watching the modified “wheel pendulum” Hans was somewhat surprised, then frowned and began to pace restlessly around the workshop, sometimes stopping so as not to crash into the moving structure. At some point, he sat down at the table and started writing something down in a hurry. Hans planned to assemble the latest version of the pendulum, which would give him answers to all his questions. The new pendulum was to consist of two identical hoops with weightless spokes. The centers of the hoops had to be rigidly connected to each other by a weightless rod with a length of l and suspended on two identical weightless threads with a length of l to one point on the ceiling (see Fig.). The radii of the hoops R, the masses m, during the movement of the hoops are in the same vertical plane. Suppose at some point the angle between the midpoint perpendicular to the rod and the vertical became equal to φ, and the rate of change of this angle was equal to $\dot{\varphi}$.

5. *(1 point)* Write down the expression for the total energy of the system at this point in time.

6. *(1 point)* Derive from this expression by taking a derivative the tangential accelerations of the centers of the hoops. Express the answers in terms of $l, R, m, g, \varphi, \dot{\varphi}$.