LPR Cup 2023

9.s04.e03

The big artist…keeps a sharp eye
on Nature and steals her tools.
Thomas Eakins

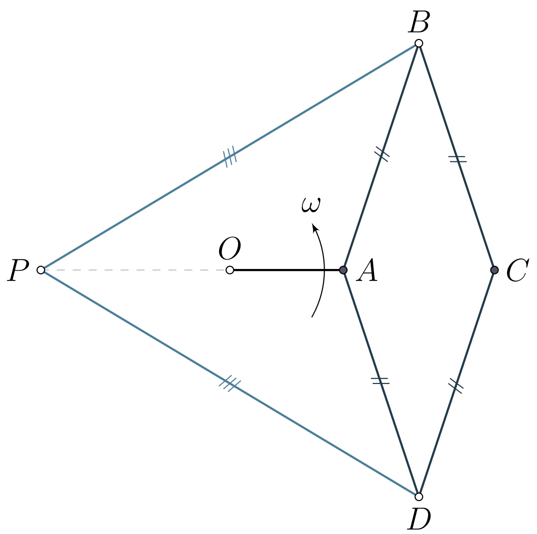
Hans' workshop

It was an early cloudy morning when Hans was approaching his workshop and saw that the door was open. When he went inside, he discovered that almost nothing had changed. Only now there was a strange mechanism on the workbench and a letter from which he learned that this was a new order that needed to be investigated and finalized.

Day I. Kinematics

Initial version of the mechanism

The mechanism consists of pivotally connected rods, shown in the figure. The lengths of the rods are such that $PD = PB = 8l$, $AB = BC = CD = DA = 4l$, $OA = d = 3l$, and the points P and O are fastened in such a way that the distance between them is $3l$. At the initial moment of time, the hinges P, O and A are on the same straight line. Hans starts the mechanism and the point A begins to rotate around a circle centered at the point O with a constant angular velocity ω. Assume that for this version and all the other versions of the mechanism the rod OA can freely intersect the rods PB and PD, and the other rods do not pass through each other.



1. *(1 point)* For how long is such movement possible?
2. *(1 point)* Find the maximum and minimum distances from the point C to the point P during this movement.

3. *(1 point)* Find the maximum and minimum velocities of the point C in the process of this movement.

4. *(1 point)* Find how the radius of curvature of the trajectory of the point C depends on time.

A modified mechanism

Hans has modified the mechanism, and now $PB = PD = a$, $PO = c$, $AO = c/\sqrt{2}$, $AB = BC = CD = DA = b$. He reset it to its original position (the points P, O and A are on the same straight line) and started the mechanism so that the point A again rotates around a circle centered at the point O with a constant angular velocity ω.

5. *(1 point)* Under what conditions can the point A make a complete revolution around the point O?

   In the following tasks of this section, assume that these conditions are met.

6. *(1 point)* Find the maximum and minimum distances from the point C to the point P during this movement.

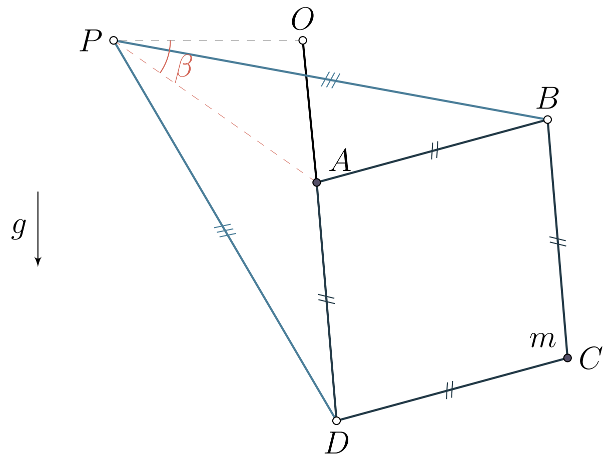
7. *(1 point)* At what moments of time is the velocity of the point C directed along the vector $\overrightarrow{PC}$?

8. *(1 point)* Find the centripetal acceleration of the point C when the angle between OP and OA is equal to $\pi/4$.

Day II. Statics

According to the order that Hans received, all the rods and hinges in the mechanism had to be weightless with the exception of the C hinge. Hans assembled a new prototype that had all these properties and fastened the points P and O on the same horizontal line at a distance of c from each other (see Fig.). The lengths of the rods in the new prototype are $PB = PD = a$, $AB = BC = CD = DA = b$, and $OA = d$.

The hinge C of mass m is much heavier than all other parts of the system.

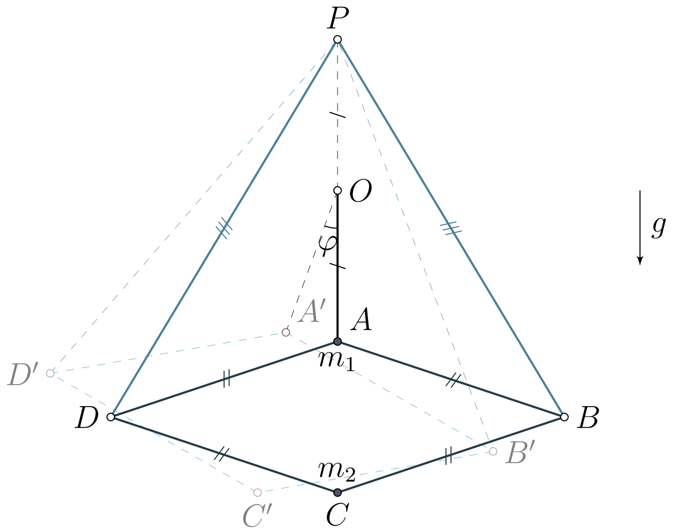


9. *(1 point)* Find all possible values of the angle β at which the prototype is in equilibrium and examine these positions for stability.

Day III. Dynamics

A couple of days before the meeting with the Customer, Hans received a letter from him with new requirements. The requirements were as follows: the hinges A and C should be massive and their masses should be equal to m_1 and m_2, respectively. Hans assembled such a prototype and fastened it at the points P and O on the same vertical line. You may assume that the remaining parts of the structure are weightless, there is no friction in the system.

Hans tilted the rod OA at a small angle φ from the equilibrium position and released it without initial velocity.



10. *(1 point)* Find the acceleration of the point A at this moment of time.

First hint — 08.05.2023 20:00 (Moscow time)
Second hint — 10.05.2023 12:00 (Moscow time)

Final of the third round — 12.05.2023 20:00 (Moscow time)