9.s04.e02

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

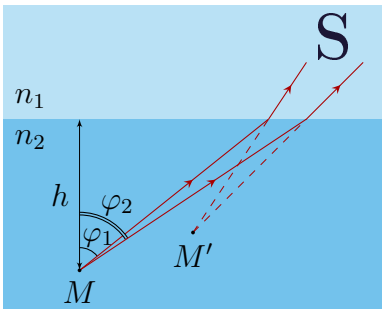
$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

   where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

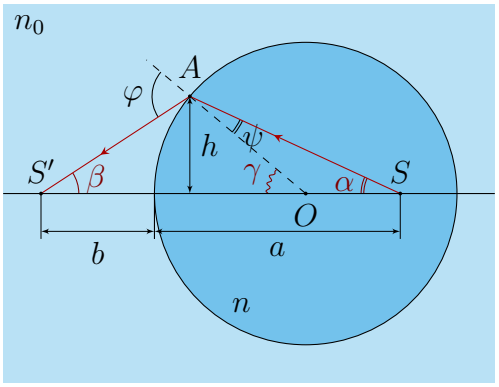
Alternative problem

Task 1. *(1 point).* Seeker stood on the bridge and peered into the surface of the water. At the bottom, reflecting the light of the gas lanterns, the coin glittered. "I need to get it. Seems like it's 70 centimeters of water here ...". Considering that Seeker has an excellent eye, but does take into account the fact that the coin is in the water, calculate the real depth of the reservoir. The refractive index of water is equal to $n = 4/3$.

Task 2. *(3 points).* Seeker saw another coin at some distance from the bridge. What distance (horizontally) separates the coin and its image? Assume that Seeker sees only the rays reflected from the coin at angles $\varphi_1 = 30°$ and $\varphi_2 = 31°$ (see fig.). The depth of the reservoir is H. The refractive index of water is equal to $n_2 = 4/3$ and the refractive index of air is equal to $n_1 = 1$.



Task 3. *(0 points)*. In a medium with a refractive index n_0, there is a Ball of radius R made of a material with a refractive index $n > n_0$. Hidden hides inside the Ball at point S. Consider a ray of light emitted by Hidden reaching the surface of the Ball at point A. After refraction, it crosses the optical axis at point S'. Let's denote the distance from S' to the nearest point of the Ball as b, and the distance from Hidden to the same point as a.



1. Express h — distance from point A to the optical axis in three different ways:
   (a) in terms of a and α,
   (b) in terms of b and β,
   (c) in terms of R and γ.
2. Write down these expressions taking into accout the fact that angles α, β, γ are small.
3. Write down Snell's law for this task taking into account the fact that angles φ and ψ are small.
4. Using all the resulting equations express $1/R$ in terms of $1/a$, $1/b$ and refractive indices.

Task 4. *(6 points)*. Seeker is located to the right of the Ball and it seems to him that Hidden is located at point S at a distance $R/2$ from the center of the Ball (see fig.). Seeker, Hidden and the center of the Ball are on the optical axis. Where might Hidden actually be hiding? The refractive index of the Ball n_2 is three times higher than the refractive index n_1 of the medium in which Seeker is located.