LPR Cup

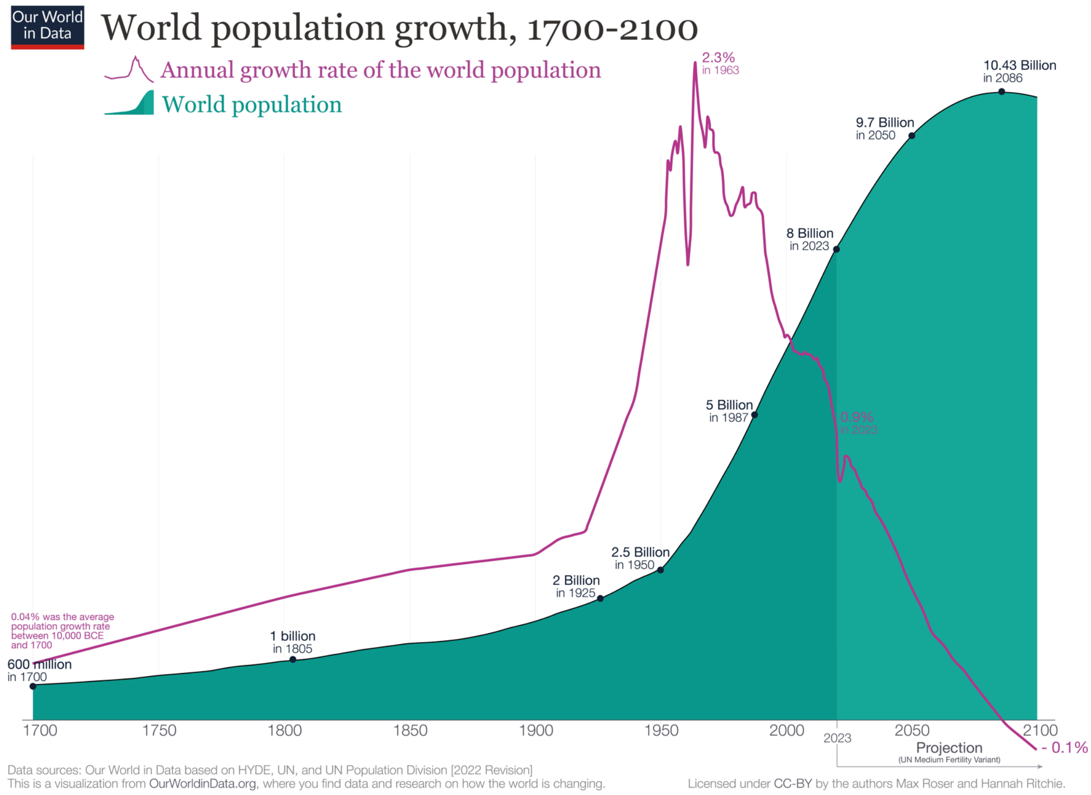
11.s05.e05

Between stimulus and response there is a space.
Viktor Frank

Synthesis

Part 1. Nuclear Reactions (1.4 points)

According to publicly available data, the 20 th century marked the beginning of rapid population growth on our planet.



1. *(0 points)* According on the graph, determine the year when the peak population growth rate occurred, i.e., the percentage of population growth for the year was maximum, and what this rate was in percent.
2. *(0.2 points)* Assuming that since 2023, the population growth rate will be equal to the peak growth rate in the 20th century, determine how many years later one person will stand on each square meter of land. Assume that the Earth's radius is 6400 km and the land area is 1/3.

Due to the rapid growth of the world's population, energy consumption is also growing rapidly. Today, humanity consumes approximately $Q = 10^{21}$ J per year. Renewable energy sources (wind, hydro, solar, etc.) can provide up to $3Q$ per year, but unfortunately, they do not solve all energy problems. The energy obtained with their help is quite expensive and requires additional conditions (wind availability, a large number of sunny days per year, etc.). The power generated by these sources varies greatly over time (for example, solar panels do not generate energy at night), which requires the creation of huge and expensive energy storage facilities.

Historically, humanity has learned to actively use natural resources: oil, coal, gas, etc. Unfortunately, their reserves are limited and unevenly distributed across the planet. For example, oil reserves are about $15Q$, coal reserves are about $150Q$. In addition, the burning of hydrocarbons leads to the emission of greenhouse gases and affects the climate.

In addition, chemical energy sources have low energy efficiency.

3. *(0.2 points)* Estimate **the energy efficiency** of burning chemical fuel, i.e., the amount of energy (in eV) per nucleon released during the combustion of one kilogram of coal ${}^{12}_{6}\mathrm{C}$. Assume that its specific heat of combustion is 31 MJ/kg.

For these reasons, humanity is searching for alternative energy sources. In the course of research in nuclear physics, it was found that the nuclei of some elements can be transformed into the nuclei of others. For example:

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}n \longrightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}n, $$

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}n \longrightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\,{}^{1}_{0}n. $$

4. *(0.2 points)* Using the concept of mass defect, estimate the energy efficiency of each reaction.

5. *(0 points)* By how many orders of magnitude is the energy efficiency of nuclear fuel greater than the energy efficiency of chemical fuel?

Nuclear fission reactions often produce unstable isotopes, the decay of which results in a chain of nuclear transformations.

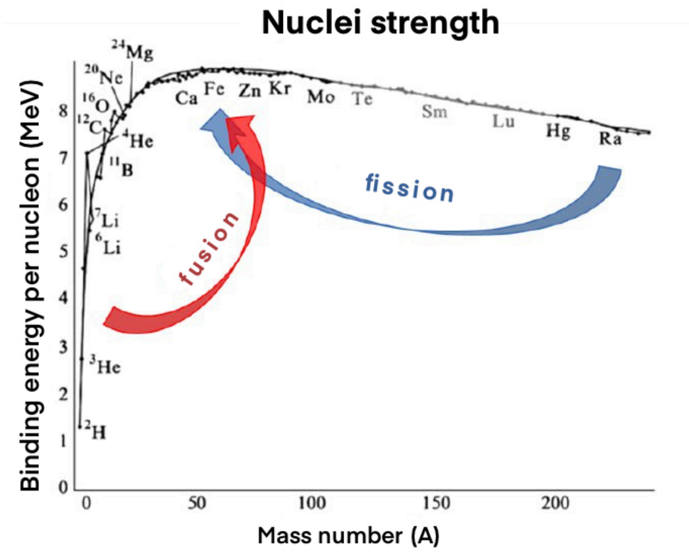
6. *(0.8 points)* Consider the possible decay chains of the nucleus ${}^{235}_{92}\mathrm{U}$, at the end of which the ${}^{207}_{82}\mathrm{Pb}$ nucleus is formed. Assume that only three types of reactions are possible: alpha decay, beta-minus decay, and proton emission. Assuming that their probabilities are related as $20 : 10 : 1$, respectively, determine the most likely set of decay products.

Note. In reality, proton emission is still an order of magnitude less likely and occurs with artificially created isotopes of chemical elements.

From all of the above, it follows that nuclear energy solves humanity's problems with energy supply. However, the operation of nuclear power plants leads to the accumulation of radioactive waste and the possibility of technological disasters, which forces us to continue the search for alternative energy sources

Part 2. Nuclear Fusion (0.5 points)

It has been established that energy can be released not only during the fission of heavy nuclei but also during the fusion of light elements from the beginning of the periodic table. A visual demonstration of this fact is shown in the figure, which depicts the dependence of the binding energy of nuclei on the atomic number. It can be seen that elements to the left of Fe tend to fuse, while those to the right tend to fission.



From a practical point of view, the following fusion reactions are of most interest:

$$
\begin{aligned}
&d + d \longrightarrow t + p + 4{,}03\ \text{MeV},\\
&d + d \longrightarrow {}^3_2\text{He} + n + 3{,}27\ \text{MeV},\\
&d + t \longrightarrow {}^4_2\text{He} + n + 17{,}6\ \text{MeV},\\
&d + {}^3_2\text{He} \longrightarrow {}^4_2\text{He} + p + 18{,}3\ \text{MeV},
\end{aligned}
$$

where d and t are the nuclei of deuterium and tritium, respectively. It is important to note that the first and second reactions are equally probable.

1. *(0 points)* Find the energy efficiency of the thermonuclear fusion reactions.

Unlike fossil fuels and nuclear fuel, deuterium is widespread in nature and is found in water. There is one deuterium atom for every 6500 hydrogen atoms.

2. *(0.5 points)* Estimate the minimum amount of gasoline that needs to be burned to obtain energy equal to that released in the fusion reaction of deuterium extracted from one glass of water (0.33 liters). The specific heat of combustion of gasoline is 44 MJ/kg.

For nuclear fusion reactions to occur, the Coulomb barrier of repulsion between interacting particles must be overcome. Therefore, for the implementation of controlled thermonuclear fusion, it is preferable to use hydrogen isotopes, which have the smallest charge among all other nuclei. However, even for them, thermonuclear fusion reactions can only proceed at very high temperatures, on the order of tens of keV (1 eV = 11604 K). At such temperatures, any substance transitions into a plasma state.

Part 3. Plasma (0.6 points)

Plasma is an ionized gas, i.e., a two-component system consisting of electrons and positively charged ions. To study the properties of this system, we will find the characteristic scales of distance and time.

Characteristic length scale

An important property of plasma is quasineutrality, i.e., the equality of positive and negative charge in any sufficiently large volume. The characteristic spatial scale in plasma is the distance at which a violation of quasineutrality is permissible.

To estimate such a distance, it is necessary to consider the screening of the potential of a point charge in a plasma. We will assume that the concentrations of electrons and ions obey the Boltzmann distribution:

$$n(r) = n_0 \exp\left(-\frac{q\varphi(r)}{kT}\right),$$

where n_0 is the normalization constant for both components, φ is the potential of the charge, q is its magnitude, and T is the temperature. **Remark.** In a plasma, the temperatures of ions and electrons may differ, but in this problem, we will assume that these components are in thermodynamic equilibrium. From the Zeroth Hint of the Third Episode, you know the Poisson equation, which has the form:

$$\Delta\varphi = -\frac{\rho}{\varepsilon_0},$$

where ρ is the charge density. In the one-dimensional case, this expression takes the form:

$$(\varphi)''xx = -\frac{\rho}{\varepsilon_0}$$

1. *(0.2 points)* Assuming that $q\varphi \ll kT$, dedimensionalize the Poisson equation and find the characteristic scale of charge separation.

Characteristic time in plasma

If the quasineutrality of the plasma is violated, the arising electric fields will tend to compensate for it.

2. *(0.2 points)* Consider a fully ionized plasma under normal conditions, in each cubic centimeter of which there are $7 \cdot 10^{16}$ electrons and ions. Suppose that there was a complete separation of charges: all the electrons gathered near one plane, and the ions near another. The distance between the planes is 1 cm. Then uncompensated charges will arise at the boundary of the region occupied by the plasma. Find the magnitude of the electric field arising in this layer.
3. *(0.2 points)* Find the time it takes for the charges to return to the equilibrium position.

When creating a fusion reactor, it is necessary to solve two main problems. The first is to confine the plasma with the required parameters for a sufficiently long time, and the second is to prevent the flow of heat and particles from damaging the reactor walls. Both of these issues are addressed through the use of magnetic fields and are analyzed in the 11th-grade problem.

In the 10th grade problem, we will get acquainted with the concept of plasma using the example of a self-sustained gas discharge, and also compare the process of maintaining the discharge and the criterion for reaching zero useful power of a thermonuclear reactor

Part 4. ~~Gorenje Criteria~~ Zero cycle (1.5 points)

This section challenges you to derive the criterion for achieving the output to zero useful power of a thermonuclear power plant.

Consider a plasma in some limited volume consisting of deuterium and tritium nuclei, as well as electrons formed during their ionization ~~(this has never happened and here it is again)~~ and having the same temperature as their entire environment ~~(not again, but once more)~~. Each type of nuclei behaves as an ideal gas with concentration n and temperature T. Deuterium and tritium undergo the fusion reaction $d + t \longrightarrow {}^4_2\mathrm{He} + n + 17.6$ MeV.

We will consider a highly simplified model in which the collision cross-section of the nuclei does not depend on their velocity. That is, two nuclei undergo a fusion reaction if they approach each other at a distance less than the known sum of some effective radii $R = r_1 + r_2$. Assume that all reaction products, without interacting with the plasma, reach the reactor walls and transfer all their energy to it in the form of heat. The reactor walls, in turn, convert this heat into electricity, which is then used to heat/maintain the plasma temperature. This feedback system operates with an efficiency of η. It is known that if the feedback mechanism and nuclear reactions are stopped, the plasma will begin to cool down with a characteristic cooling time τ, which is determined by the design features of the reactor and is one of its key parameters.

1. *(1.5 points)* Under what circumstance on the value of $n\tau$ is it possible for the reactor to operate in such way that the described system can operate for an arbitrary amount of time and be "self-sustaining"due to the feedback mechanism?

Part 5. Fusion installations (6 points)

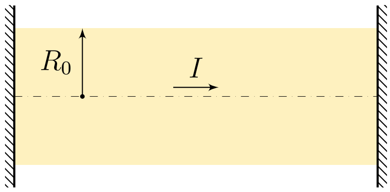
It is known that in the universe, thermonuclear reactions occur in the interiors of stars at high temperatures on the order of 10^7 K, where the plasma is confined by gravitational forces (the masses of stars are very large, for example, the mass of the Sun is approximately 10^{30}kg, therefore the gravitational forces are enormous and sufficient to confine the plasma). While creating a device that carries out thermonuclear fusion in laboratory conditions, the first problem we encounter is confining the plasma in a limited volume. The second problem is how to reduce the heat and particle fluxes reaching the reactor walls.

In this problem, we will analyze the first of these two problems.

Z-pinch

Stationary Equilibrium

Consider a plasma discharge in the form of a cylinder of length l and radius R_0 (see figure).

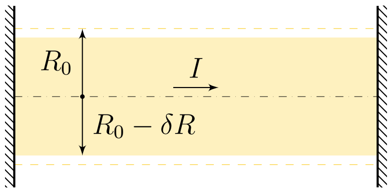


1. *(0.2 points)* What current must flow through the plasma for it to be held in equilibrium by its own magnetic field?

Assume that the plasma is in a stationary state, its temperature is T, and the number of ions per unit length is N.

Equilibrium Stability

Consider a situation where a uniform compression of the plasma column has occurred along its entire length (see figure).



2. *(1 point)* Analyze the stability of the equilibrium position from the previous point. For a stable equilibrium position,

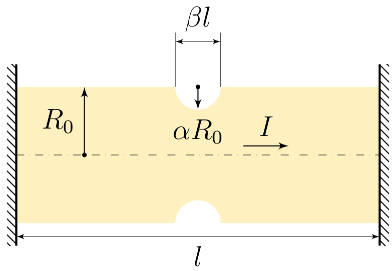
find the period of oscillations. Consider the process in isothermal and adiabatic approximations.

The plasma temperature is T, the number of ions per unit length is N, the initial radius of the cylinder is R_0, the ion mass is m_i, and the electrons mass is $m_e \ll m_i$.

Note: Assume that the radial component of the particle velocities associated with the oscillations is proportional to the distance to the axis.

Shape Instability

At some point, a constriction appears on the plasma column with a characteristic length of βl and depth of αR_0 (see figure).



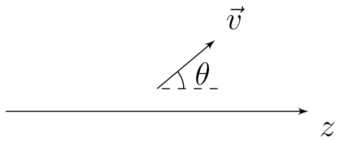
3. *(0.2 points)* For what values of β will the constriction continue to contract towards the center and break the plasma column?

Consider α and β to be known.

Magnetic Mirrors

Motion of a Particle in an Inhomogeneous Field

A particle of mass m carrying charge e moves in a magnetic field whose z-component depends only on the z coordinate: $B_z = B_z(z)$. The magnitude of the particle's velocity is v and is directed at an angle θ to the z axis. The field along the z axis varies slowly enough $(\frac{dB_z}{dz} \ll \frac{eB^2}{mv})$.

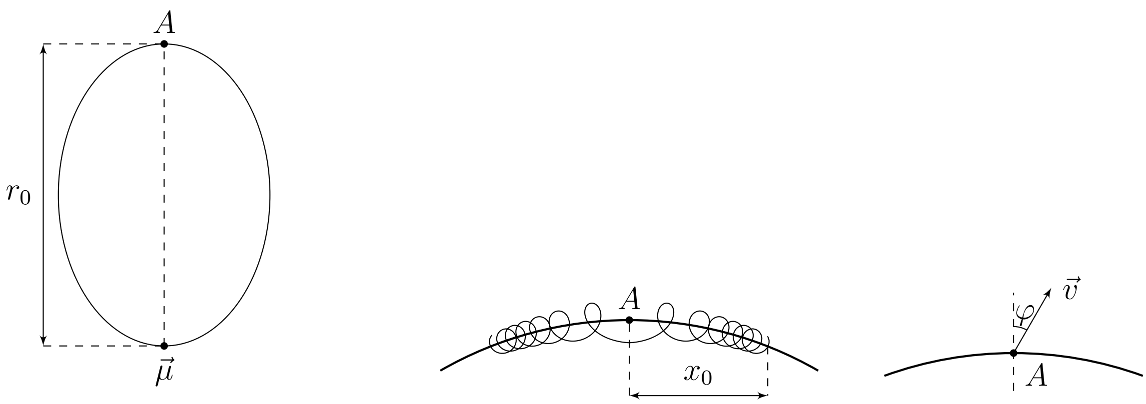


1. *(0.2 points)* Find the time-averaged force F_z exerted on the particle by the magnetic field in the vicinity of the coordinate z.

2. *(0.1 points)* Using the previous point, find the invariant of motion of the particle.

Confinement in a Dipole Field

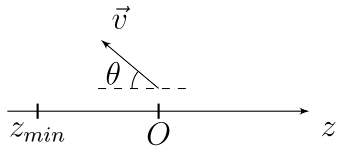
Consider the motion of a charged particle along a magnetic dipole field line in the vicinity of point **A**, located at a distance r_0 (see figure). Initially, the particle at point **A** has a velocity directed at an angle φ to the normal of the field line.



3. *(0.3 points)* Find the amplitude of the particle's oscillations x_0.

0.0.1 Let's Move It

First A particle of mass m carrying a charge e moves in a magnetic field whose z-component depends only on the coordinate z: $B_z = B_0 e^{-kz}$, $k > 0$. The magnitude of the particle's velocity is v and is directed at an angle $0 < \theta < \pi$ against the z axis (see figure). The field along the z axis changes slowly enough ($k \ll \frac{eB}{mv}$). At the initial moment of time, the particle is at $z = 0$.



4. *(0 points)* Show that the radial component of the field is small.

5. *(0.3 points)* Let us denote the minimum z-coordinate that the particle reaches during its motion as z_{min}. Find z_{min} and the time t of the particle's motion from 0 to z_{min}.

Second Consider a situation in which the field decays exponentially (e.g., when the source is switched off) according to the law $B_0 = Ae^{-\alpha t}$, $\alpha > 0$, the dependence on the z-coordinate remains the same: $B = B_0 e^{-kz}$.

6. *(0.1 points)* Find the relationship between α, k, v, and θ at which the particle will move indefinitely to the left against the direction of the z axis.

7. *(0.1 points)* Find the drift velocity of the particle after a long period of time.

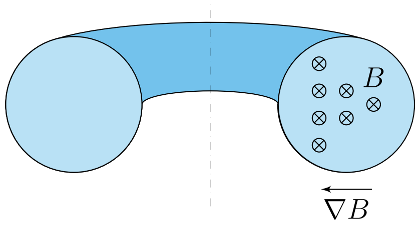
8. *(0.4 points)* Find z_{min} and t in the laboratory frame of reference, defined similarly to the terms in part **First**, for the case of $\frac{\alpha}{k} \ll v$ (this condition will allow you to simplify the general form of the expression).

9. *(0.1 points)* For the same values of k and θ, $\frac{\alpha}{k} \ll v$, which of the values of z_{min} is smaller: from **First** or from **Second**?

Tokamak

Drift in an alternating field B

A particle is in a toroidal trap (see figure) in an inhomogeneous field B. The velocity component perpendicular to the field is $v_\perp$.

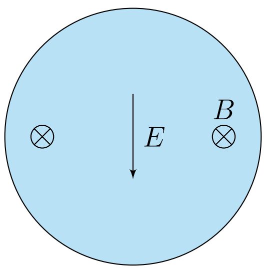


1. *(0.5 points)* Find the drift velocity of the plasma particles. Will the plasma be ejected onto the walls due to the presence of this drift velocity in the particles? Justify your answer.

Consider the field B, the gradient of the field modulus $\nabla|B|$, the mass of the particle m, the charge of the particle q, and the perpendicular velocity $v_\perp$ to be known.

0.0.2 Drift in crossed fields E and B

A particle is in a toroidal trap (see figure) in crossed fields B and E.



2. *(0 points)* Find the drift velocity of the particle. Will the plasma be ejected onto the walls of the trap due to the presence of this drift velocity in the particles? Justify your answer.

Consider the field B, the field E, the mass m, and the charge of the particle q to be known.

Helical Field Lines

To avoid plasma ejection, the field lines are "twisted". Since the particles move around the field lines, this "twisting"causes the particles to follow the field lines and be located sometimes on one side and sometimes on the opposite side of the trap's center. Due to this, the drift is compensated on average: on the one hand, the particles drift towards the center of the trap, on the opposite side they drift away from the center.

The twisting is performed as follows: the field along the annular axis of the torus (poloidal field) is created by a toroidal coil (toroidal current), the field around the annular axis of the torus (toroidal field) is created by the current inside the plasma (poloidal current). Together, these fields produce field lines in the form of helical lines along the surface of the torus.

Unfortunately, not every configuration of field lines is stable. Let the field line be a helix (along the surface of the torus) with a pitch h. To ensure the stability of the plasma column, the maximum wavelength of the perturbation λ must be less than the pitch of the helix h. In this case, the wavelength of the perturbation in the torus cannot exceed the length of the system, i.e. $2\pi R$.

Then the stability condition for helical perturbations can be written as:

$$2\pi R \leq h,$$

where R is the major radius of the torus, h is the pitch of the helix.

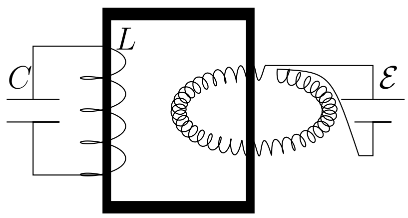
The ratio of these lengths

$$q = \frac{h}{2\pi R} \geq 1$$

is called the safety factor.

3. *(0.5 points)* Find the relationship between the field created by the toroidal coil B_z and the field created by the current in the plasma B_φ, if the safety factor is q and the radius of the plasma column is a. Draw a diagram of the field lines.

Tokamak



A tokamak (see figure) is the most promising system for controlled thermonuclear fusion. To confine the plasma in a tokamak, the magnetic field of a toroidal coil made of copper wire and the field created by the eddy current in the plasma are used. The current in the toroidal coil is generated by a direct current source with EMF $\mathcal{E}$. The current inside the plasma is created by means of an alternating magnetic field inside a ferromagnetic core (inductor), which passes through the center of the torus and through an external circuit consisting of a capacitor C and a coil L (see figure).

Assume that the system is stable against helical perturbations with $q \geq 1$.

4. *(2 points)* Find the minimum EMF $\mathcal{E}$ of the source, as well as the breakdown voltage for which the capacitor must be designed.

The conductivity of copper is $\sigma_{\text{copper}} = 57$ MS/m, the plasma temperature is $T = 2 \cdot 10^7$ K, the cross-sectional area of the copper wire of the toroidal coil is $S = 1$ mm^2, the distance from the center of the generating circle to the axis of rotation of the torus is $R = 1.5$ m, the radius of the generating circle is $r = 15$ cm, the ion concentration in the plasma is $n = 10^{20}$ m^{-3}, and the number of turns of the coil in the external circuit is $N = 1000$.

Assume that the gap between the plasma column and the wall of the torus is 3 cm.

At $T = 2 \cdot 10^7$ K, the plasma conductivity is equal to the conductivity of copper and continues to increase with temperature as $T^{3/2}$.

You can assume that the magnetic pressure on the surface of the plasma column is an order of magnitude higher than the plasma pressure. The induced current inside the toroidal coil and the inductance of the plasma column can be neglected.

First hint — 29.05.2024 20:00 (Moscow time)
Second hint — 31.05.2024 12:00 (Moscow time)

Final of the fifth round — 02.06.2024 18:00 (Moscow time)