



LPR Cup 2023

11.s04.e04

*Life has ended like a film,
A child's cry, and we start over...
band «Дороги меняют цвет»*

Phenomenon

In this problem, you should investigate the equilibrium configurations of plane systems with a fixed perimeter under the influence of surface-tension forces. In all parts of the problem, consider all films to be **single-sided** and light. The coefficient of surface-tension of a film, surrounding a quadrangle, is σ . Also, in all parts of the problem, the gravitational field can be neglected.

Equilibrium of a quadrangle in a film

The quadrangle $ABCD$ with sides $AB = a$, $BC = b$, $CD = c$ and $DA = d$ consists of jointed rods. The quadrangle is surrounded by a soap film that does not leak inside it.

1. (1 point) Let $a = 3l$, $b = l$, $c = d = 2l$. Find the area of the quadrangle in the equilibrium position with an accuracy of 1%.
2. (1 point) While holding the quadrangle from the previous part in the equilibrium position, a film with a surface tension coefficient of 2σ was added inside the figure. The figure was released with a small random disturbance. Find the possible values of the area of the figure in the new equilibrium position.
3. (3 points) For arbitrary a , b , c , d and without the film inside, find the angle ABC of the quadrangle in the equilibrium position.

Oscillations of the quadrangle

The weightless rods AB and BC , as well as BC and CD are connected sequentially by joints B and C , respectively, of mass m each. The ends of the rods A and D are fixed pivotally on the ends of a stationary spoke. The film does not leak in the area between the rods and the spoke. $AB = 4l$, $BC = CD = AD = 2l$.

4. (1 point) Find the angle BAD at the equilibrium position of the system.
5. (3 points) Find the period of small oscillations of the system near the equilibrium position.
6. (1 point) While holding the quadrangle in the equilibrium position, a film with a surface tension coefficient of 2σ was added inside the figure once again. The figure was released with a small random disturbance. Find the possible values of the area of the figure in the new equilibrium position.

First hint — 15.05.2023 20:00 (Moscow time)

Second hint — 17.05.2023 12:00 (Moscow time)

Final of the fourth round — 19.05.2023 20:00 (Moscow time)