LPR Cup 2023

11.s04.e02

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

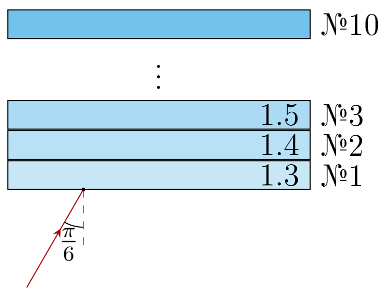
$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

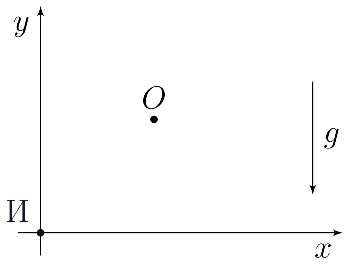
Alternative problem

Task 1 *(0 points)* There is a system of plane-parallel plates, in which the refractive index of each following plate is 0.1 greater than the refractive index of the previous one (see Figure). The refractive index of the first plate is 1.3, and a ray from the air hits it so that the angle of incidence is $\pi/6$. Find the angle between the ray and the normal line in the tenth plate.



Task 2. *(1 point)* The refractive index of the stratified medium varies according to the law $n(y) = n_0 - \alpha y$, where α and n_0 are known positive constants. Hidden is located at the point with the coordinate $y = 0$ and emits a beam of light at an angle pi/6 to the y axis. At what maximum depth $y_{\max}$ can the beam penetrate into the medium?

Task 3. *(0 points)* Seeker is throwing stones at the window. Express the equation of the trajectory of the stone in $y(x)$ coordinates in terms of the initial velocity of the stone v_0, the angle at which Seeker throws the stone ϕ_0, and the free-fall acceleration g.



Task 4. Seeker is located at the origin of the coordinate plane and is throwing stones at a target, which coordinates are $(9; 9)$ m. The initial speed of the stone is 15 m/s.

1. *(0,5 points)* At what angle should Seeker throw stones to hit the target?
2. *(0,5 points)* How long does the flight of the stone from Seeker to the target last?
3. *(2 points)* Seeker began to get tired and the initial velocity of the stones began to decrease. What is the minimum speed of a stone, at which Seeker can still hit the target?

Task 5. *(0 points)* A ray of light passes 1 m in one case in a medium with a refractive index of 1.5, and in another case – in a medium with a refractive index of 1.2. How much do the time intervals of passing this distance differ in these media?

Task 6. *(1 point)* Hidden is located at the origin in a stratified medium, the refractive index of which varies according to the law $n(y) = n_0 - \alpha y$, where α and n_0 are known positive constants, and emits a beam of light along the y axis. Find the time, which takes for the beam to hit the point with the coordinate $1/\alpha$. Consider $n_0 > 2$.

Task 7. Hidden is located at the origin in a stratified medium. He emitted a ray, so that its trajectory has the form:

$$y(x) = 1 - \left(e^{2x/\sqrt{3}} + 3e^{-2x/\sqrt{3}}\right)/4.$$

1. *(2 points)* Find the tangent of the initial angle between the ray and the horizontal axis.
2. *(3 points)* Find the function of the refractive index of the medium $n(y)$. Assume $n = n_0$ for $y = 0$, where n_0 is a known constant.