



LPR Cup

11.s01.e06

Hint 1



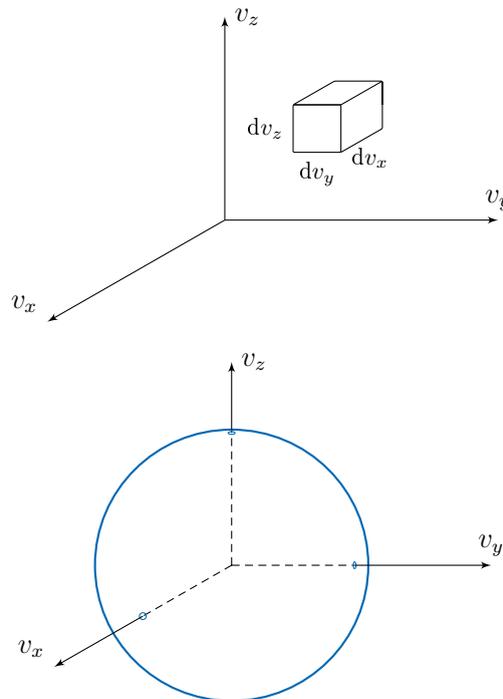
Hint 1

As a first hint, a brief Maxwell distribution theory will be used.

We denote by dN_v the average number of ideal gas molecules having velocity components lying between v_x and $v_x + dv_x$, v_y and $v_y + dv_y$, v_z and $v_z + dv_z$. It can be proved that the following equality is true:

$$dN_v = N \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left(-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT} \right) dv_x dv_y dv_z, \quad (1)$$

where N — the total number of molecules, m is the mass of one molecule, k is the Boltzmann constant, T is the gas temperature. The physical meaning of this expression is that if we select a small volume $dv_x dv_y dv_z$ in the velocity space, then dN_v will be equal to the number of gas molecules for which the velocities belong to this small volume. In other words, the value dN_v/N is equal to the probability that a randomly selected molecule has a velocity at which the projections belong to the intervals $[v_x, v_x + dv_x]$, $[v_y, v_y + dv_y]$ and $[v_z, v_z + dv_z]$. The formula (1) is called the Maxwell distribution.



Note that this is not the only way this distribution can be written. For example, you can write an expression for the probability that the absolute value of the velocity will belong to the interval $[v, v + dv]$. In this case, they say that we are moving from a distribution over projections of velocities to a distribution over absolute values of velocity. In the space of velocities, this is equivalent to the fact that instead of the elementary volume $dv_x dv_y dv_z$ we consider a thin-walled sphere of radius v and thickness dv (see. Fig.). Replacing the elementary volume $dv_x dv_y dv_z$ with the volume of the thin-walled sphere $4\pi v^2 dv$ we obtain a distribution over the absolute values of the velocities:

$$dN_v = N \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left(-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT} \right) 4\pi v^2 dv. \quad (2)$$

We write another version of the Maxwell distribution — the Maxwell energy distribution. We denote the kinetic energy of an individual molecule by

$$\varepsilon = \frac{mv^2}{2}.$$

Differentiating the last equation and multiplying it by the speed of the molecule v we get:

$$v^2 dv = \sqrt{\frac{2\varepsilon}{m}} \frac{d\varepsilon}{m}.$$

Whence, using (2), we obtain the Maxwell energy distribution:

$$dn_\varepsilon = N \frac{2}{\sqrt{\pi}(kT)^3} e^{-\frac{\varepsilon}{kT}} \sqrt{\varepsilon} d\varepsilon.$$