Leonard: Why don't you just tell people you're a zebra?
Sheldon: Well, why don't you just tell people you're one of the seven dwarves.
Leonard: Because I'm Frodo.
Sheldon: Yes, well, I'm the Doppler Effect.

"The Big Bang Theory"

Coda

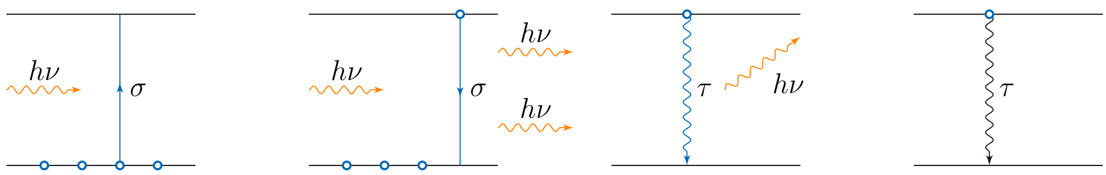
This problem is devoted to such a section of physics as the interaction of laser radiation with matter. All necessary facts from the theory will be given in the first section. If you have any questions about this section, then you can contact the jury, maybe they will answer something different from « no comments », because this section of physics is beyond the scope of the school curriculum.

Good luck!

What can electrons do?

In atoms, electrons are located at energy levels. In this problem, we will consider only discrete level systems and for a start we will study the possible types of electron transitions between them.

Consider the two-level system shown in the figure. The lower energy level will be called the ground state, the upper level – excited state, the difference between the energy levels will be denoted by ΔE.



а) Absorption

б) Stimulated emission

в) Spontaneous emission

г) Radiationless de-excitation

We can assume that under the action of laser radiation on an atom, atom interacts with individual quanta of the electromagnetic field, which are called photons. The energy of an individual quantum is $h\nu$, where ν is the frequency of electromagnetic radiation. In the framework of this problem, we assume that a photon can interact with an atom only if its energy is ΔE (note that exact equality is not necessary, because even an isolated atom has a natural line width δE, however, this fact is not taken into account in this problem).

Two variants of the interaction of an atom and a photon are possible. The first option is *photon absorption* (Fig. 1a). An electron in the ground state can absorb a photon with some probability. The probability of absorption of an individual photon by a separate electron is characterized by *absorption cross section* σ. The dimension of the absorption cross section is cm^2 (this value can be visualized as a certain electron size for a photon, similar to how the characteristic size of molecules was introduced in the problems of molecular kinetic theory, for example, when studying the mean free path). Similarly, an electron located at an excited level can go into the ground state due to the action of electromagnetic radiation, and to emit a photon that will be exactly the same (direction, frequency, initial phase, etc.) as the photon, which has «dropped» the electron down. We will call such a process *stimulated emission*. It is characterized by a *photon emission cross-section*, which is exactly equal to the absorption cross-section.

In addition to transitions due to the interaction of radiation with atoms, other transitions are possible, which are called *relaxation*. If the electron is in an excited state, then at some moment it will go into the ground state. In this case, this process can be accompanied by emission of a photon of frequency ν in a random direction, the so-called *spontaneous emission* (Fig. 1c) or can occur non-radiatively when the electron energy goes into heat (Fig. 1d). In the framework of this problem, we will neglect spontaneous emission and consider only the nonradiative version of excitation relaxation.

Let us pass to a quantitative description of the rate of change of the populations of electrons in the ground and excited states.

For absorption and stimulated emission of photons, it is easy to understand what the number of transitions per unit time is equal to. Let n_1 and n_2 be the electron concentrations (i.e. the number of electrons at the corresponding level in $1\,\mathrm{cm}^3$ of substance) in the ground and excited states, respectively (hereinafter, we will characterize the populations of levels by electron concentration). The light intensity I is equal to the radiation energy passing per unit time per unit area ($\mathrm{J}/(\mathrm{s}\cdot\mathrm{cm}^2)$) and is equal to $h\nu F$, where F is the photon flux density (Pieces/($\mathrm{s}\cdot\mathrm{cm}^2$)). It is clear that the number of transitions per unit time due to photon absorption is proportional to the photon flux density and electron density. The proportionality coefficient is exactly equal to the absorption cross section (this fact is clear due to dimensional considerations). In other words, if we consider **only** the process of absorption of photons, then the following equality is true:

$$\begin{cases} \dot{n}_1 = -F\sigma n_1 \\ \dot{n}_2 = F\sigma n_1 \end{cases}$$

Similarly, we can obtain the rate of change of level populations in case of stimulated emission:

$$\begin{cases} \dot{n}_1 = F\sigma n_2 \\ \dot{n}_2 = -F\sigma n_2 \end{cases}$$

For a quantitative description of relaxation processes in an atom, characteristic times are introduced for a particular process (spontaneous emission, nonradiative relaxation, etc.). We denote the characteristic time of nonradiative relaxation τ. If the electron is in an excited state and we consider only the nonradiative relaxation process, then the rate of change of the level population will be determined by the relation:

$$\dot{n}_2 = -\frac{n_2}{\tau}$$

1. In the ground state at the initial moment of time there are N electrons (where N is the concentration), there are no electrons in the excited state. Permanent laser radiation with a photon flux density of F begins to act on the system. Find the dependence of the level populations in the ground and excited states on time (consider only stimulated transitions). Plot them on one chart. Consider the absorption cross section σ as known. *(0,5 points)*

2. Suppose that N_2 electrons are in an excited state at the initial moment. Get the dependence $n_2(t)$, considering only nonradiative transitions. The time of nonradiative relaxation τ is known. *(0,25 points)*

Saturable absorption

Consider a two-level system on which laser radiation begins to act with a constant photon flux density F. The absorption cross section σ and the nonradiative relaxation time τ are known. At the initial moment of time, all electrons are in the ground state, i.e. $n_1(0) = N$, $n_2(0) = 0$.

1. Write down a system of equations that will describe changes in the populations of the levels of such a system(consider both: stimulated and nonradiative transitions). (0,5 points)
2. Starting at a certain point in time, we can assume that the number of transitions per unit time from the ground state to the excited state will be equal to the number of transitions per unit time in the opposite direction, i.e. level populations do not change. In this case, they say that *stationary distribution* of electrons over the levels has been established. Find the dependences of $n_1(F)$ and $n_2(F)$ in the stationary state. Draw these dependencies on one chart. (0,5 points)
3. The absorption coefficient of a substance is the quantity:

$$\alpha = n_1\sigma - n_2\sigma.$$

   Plot the dependence of the absorption coefficient on photon flux density in case of stationary distribution. (0,25 points)

4. From dimensional considerations, it can be understood that the change in the photon flux density during radiation propagation in matter is described by the equation:

$$\frac{\mathrm{d}F}{\mathrm{d}z} = -\alpha F.$$

   (a) Find in case of stationary distribution what the transmission of a medium with a thickness of h is equal to, i.e. the value $T = F_{out}/F_0$, where F_0 is the photon flux density at the entrance to the medium, F_{out} is the photon flux density at the exit from the medium, in case of a low photon flux density. Write the expression using N, F, σ, τ и h. (0,5 points)

   (b) Get the transcendental equation that relates the transmission and flux density at the entrance to the medium for an arbitrary photon flux density. Get the dependence on T, N, F, σ, τ и h. (0,75 point)

Doppler broadening

In the case of gaseous media, absorption of laser radiation at a frequency of $\nu \neq \Delta E/h$ is possible, which is due to the Doppler effect. Indeed, let the frequency of the laser radiation be not equal to the resonant frequency $\nu_0 = \Delta E/h$. Due to the fact that atoms are in disordered motion in gases, for an atom whose projection of the velocity on the direction of propagation of laser radiation is V_x, the frequency of laser radiation will be $\nu_{at} = \nu\,(1 - V_x/c)$. In that case, if $\nu_{at} = \nu_0$, the laser radiation will be absorbed.

1. Using your knowledge about the Maxwell distribution, obtain the dependence of the absorption coefficient on the frequency of laser radiation and plot the dependence of the absorption coefficient on frequency. (1 point)

2. At what frequency of laser radiation will the absorption coefficient be half the maximum? Give the answer in terms of ν_0, T (temperature), m (mass of an individual atom), c (speed of light). (0,75 points)

3. Make a numerical estimate of the line width $(\nu - \nu_0)$ for the sodium D-line. The resonant frequency is one of the transitions in this line $5.1\,10^{14}$Hz, $M = 23$, consider that the gas temperature is 500K. (0,75 points)

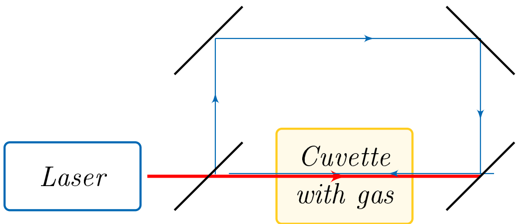
4. How will the plot of the dependence of the spectral density of the absorption coefficient on frequency from the previous question change when the photon flux density changes? Present the answer in the form of graphs for three different photon flux densities («small», «medium», «big»). Think about what is the characteristic photon flux density with which you need to compare the photon flux density of laser radiation. (0,5 points)

Many of you may do not know the Maxwell distribution. In this case, you can study it in literature. Note that for the further solution of the problem, it is enough to understand how the dependence looks, and its exact expression is not necessary.

Experimental study

Two-level scheme

Consider a gas that is characterized by a system of two levels. Let's put the gas in a transparent cuvette and consider the experimental setup shown in the figure. Powerful laser radiation (red line) first hits the dividing plate. The main part of the laser radiation is distributed in the same direction (*pump pulse*) and only a small part is reflected (*trial beam*, blue line). As a source of radiation, a laser is used that can smoothly adjust the frequency of radiation.



1. Let the frequency of laser radiation be less than the resonant frequency of the electron transition between levels $\nu < \nu_0$. We will direct the X axis in the direction of propagation of powerful laser radiation. Find the projection of velocities of atoms that will absorb the powerful pump radiation and the projection of velocities of atoms, which will absorb the test beam. (0,5 points)

2. Draw a graph of the dependence of the number of electrons in the excited and ground state on the projection of the atom's velocity on the X axis. The graph can be adjusted to the maximum value (two graphs with 0,5 points each).

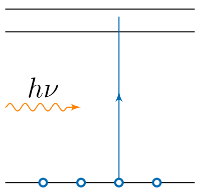
3. Give the answer to the previous two points for the case when $\nu = \nu_0$. (0,25 points)

4. Make a numerical estimate of the line width $(\nu - \nu_0)$ for the sodium D-line. The resonant frequency is one of the transitions in this line $5.1\,10^{14}$Hz, $M = 23$, consider that the gas

temperature is 500K. (0,75 points)

Three-level scheme

One of the main problems of spectroscopy is the resolution of two closely lying excited levels (see figure). In this case, due to the Doppler effect, when the atoms are excited by laser radiation, the absorption lines merge, because for any frequency of electromagnetic waves, there are electrons that have a suitable projection of the speed of the atom's movement, so that there is a transition to either the first or second excited level. To solve this problem, the experimental setup from the previous paragraph was proposed.



Let's denote the resonant frequency of transition from the ground state to the first excited state as ν_1, and the frequency of transition from the ground state to the second excited state as ν_2. For certainty, we will consider $\nu_2 > \nu_1$. The absorption sections for these transitions will be considered the same.

1. Plot dependence of the spectral density of the absorption coefficient of the test pulse on frequency. (1,5 points)

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First hint — 01.06.2020 16:00 (Moscow time)
Second hint — 03.06.2020 16:00 (Moscow time)

Final of the sixth round — 05.06.2020 23:59 (Moscow time)