

LPR VI Cup

10.s06.e03

*It is like I am an Egyptian,
With the Sun and the heat by my side,
With its claws dark sky is itching
Of a gentle Sphinx, in the shadows behind.
Picnic. The Egyptian*

Sphinx

It seemed that even the air here was melting from the heat. Seeker ran his tongue over his parched, cracked lips and tasted a metallic, bitter flavor in his mouth. It was unbearably stuffy, and the persistent, inviting shouts of the merchants, together with the chatter of the buyers in the bazaar, merged into a monotonous and unpleasant hum. The endless bustle and haggling, which seemed to hang in the air like a gray, thick, and dark substance visible only to Seeker, made this place unbearably sticky and viscous, and he felt it like a second skin he could not shed.

Pulling the hood of his tattered, sun-bleached cloak deeper over his head, he made his way through the crowd, trying to minimize contact, which at times brought him painfully sharp sensations. Seeker had no idea how long he had been searching, but by his internal clock, it had already been three days without sleep and with no bearings except for the Shadow—something that simply could not be here at this time of day, yet which continued to lead him on, clearly with no intention of stopping.

Entering a dark alley, Seeker was momentarily afraid he was about to lose his only lead, but that did not happen. Shadow, as if nothing had happened, continued moving and slipped behind one of the inconspicuous doors. Seeker pushed it open and found himself inside.

It was chilly here. Seeker walked to the center of a small hall, along the right wall of which stretched shelves filled with books in green bindings adorned with gold embossing, while on the left hung a tapestry—oddly out of place in this room—depicting sleeping lions, grazing bulls, and birds soaring in the sky. Seeker shuddered when he saw, at the far end of the room, mirrors with which he had always had a complicated relationship.

The heavy door slammed shut with a dull thud, swallowing the noisy bustle of the bazaar. Seeker spun around and froze: before him stood a woman with the bearing of a lion and the profile of an eagle. Her unblinking, lashless eyes stared piercingly at him. On her right arm dozed a hairless cat, which she absentmindedly stroked with slender fingers.

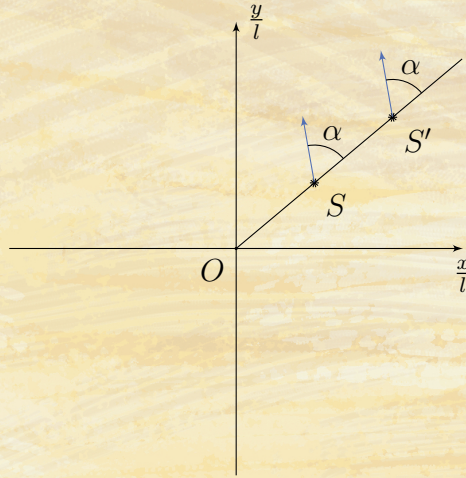
“If you leave now, you will never find it. If you answer incorrectly, you will lose what you have found,” she said.

First riddle

In a medium with an index of refraction

$$n(x, y) = \frac{n_0}{\sqrt{\left(\frac{x}{l}\right)^2 + (1 - a^2) \left(\frac{y}{l}\right)^2}}, \quad \text{where } a \in [0, 1],$$

there is a light source at a point S with coordinates $\left(\frac{l}{2}, \frac{\sqrt{3}l}{2}\right)$, and the another one at a point S' with coordinates $(l, \sqrt{3}l)$. We will consider the paths of rays, which were emitted by the light sources at an angle α to the line SO , where O is the origin, $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (see the figure below).



We introduce polar coordinates:

$$\begin{cases} \frac{x}{l} = r \cos \varphi, \\ \frac{y}{l} = r \sin \varphi, \end{cases} \quad \varphi \in (0, 2\pi], \quad r \in [0, \infty)$$

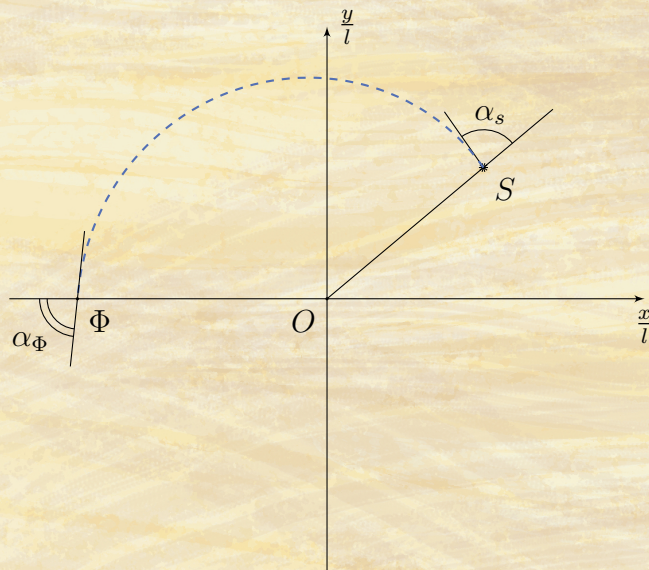
1. (1 point) Paths of the rays intersect the x axis at points Φ and Φ' . Find the ratio $\frac{O\Phi'}{O\Phi}$.
2. (1 point) Find the rays' paths for $\alpha = \frac{\pi}{2}$.
3. (6 points) Determine the dependence of the maximum polar angle φ_{\max} for the path of the ray, emitted from S at an angle α .

Second riddle

The roots were removed from the medium, and now the index of refraction takes the form:

$$n(x, y) = \frac{n_0}{\left(\frac{x}{l}\right)^2 + (1 - a^2) \left(\frac{y}{l}\right)^2}, \quad \text{for some } a \in [0, 1].$$

We still consider a ray emitted at an angle α_S to the line SO from the point S with coordinates $\left(\frac{l}{2}, \frac{\sqrt{3}l}{2}\right)$ (see the figure below). The parameter a and angle α_S are such, that the light path contains a point Φ with coordinates $(-2l, 0)$. The angle between the path and the line $O\Phi$ is equal to α_Φ .



4. (2 points) Find the optical path length for the segment of the path $S\Phi$.

Zero hint — 11.05.2025 15:00 (Moscow time)

First hint — 12.05.2025 20:00 (Moscow time)

Second hint — 14.05.2025 12:00 (Moscow time)

Final of the third round — 16.05.2025 20:00 (Moscow time)