LPR VI Cup

10.s06.e01

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

   where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative task

While trainings, which were described in the main problem, Ant-Traveller suddenly discovered that he had long been walking along the edge — in the most literal sense of this word. Not without resistance, but still, his inner adventurist for a time handed over the reins to a pragmatic rationalist (what won't you do for profitable acquaintances?), and for safety reasons he climbed out the aquarium walled in on all sides and pushed it farther away from the edge of the step he had reached during his training.

Thinking a little, our hero decided to fix it to the surface. "Out of harm's way,"thought Little Ant and continued his trainings.

Part 1. For Safety Reasons

A fixed and fully enclosed aquarium, shaped as a rectangular parallelepiped, is placed on a horizontal surface and filled to the brim with a liquid of density ρ_{liq}. The masses of the liquid and the aquarium body are M_{liq} and M_{aq}, respectively. The volume of the bathyscaphe is V, its average density (including Ant) is ρ_{b}, and it is located near the center of the left wall of the vessel.

The height of the vessel is $2h_0 + h_{\text{b}}$, and its length is $S + S_{\text{b}}$, where h_{b} and S_{b} are the dimensions of the bathyscaphe[1]. Assume that in all parts, the size of the bathyscaphe is not negligible.

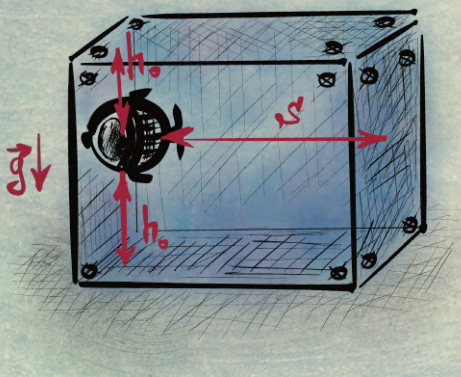
At a certain moment, the bathyscaphe begins to move horizontally toward the opposite wall. Find:

1. *(1 point)* The displacement of the center of mass of the vessel with water when the bathyscaphe touches the opposite wall of the vessel.

Part 2. A slippery story

Noticing some differences in controlling «Denise», Ant became very intrigued and decided to try placing the vessel on a frictionless surface.

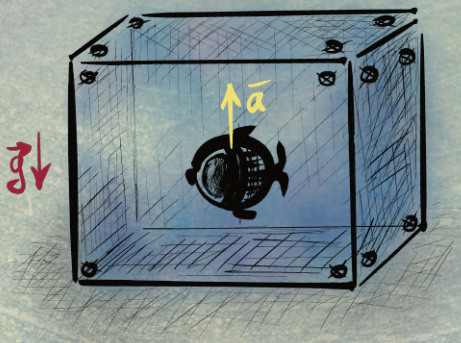
He returned the bathyscaphe to its initial position relative to the aquarium walls, but now the aquarium itself is no longer fixed and rests on a smooth horizontal surface. Ant suddenly gains a horizontal velocity v.



1. *(2 points)* What will be the vessel's velocity of moving across the floor?

[1]The distances from the very top point of the bathyscaphe to the top face of the aquarium and from the very bottom point of the bathyscaphe to the bottom face of the aquarium are both h_0, and the distance from the very right point of the bathyscaphe to the opposite wall is S.

Part 3. Laying Low



As the final experiment was nearing its completion, someone knocked on our adventurer's door, and he remembered the dubious methods he had used to obtain the sum needed to purchase the bathyscaphe.

«I need to lay low ~~in Bruges~~»—thought our hero. He positioned the bathyscaphe near the center of the lower edge of the vessel and hid. ~~Years passed~~ Soon, the knocking stopped, but Little Ant continued to anxiously wait for the sound of retreating footsteps. For a while, nothing ~~seemed amiss~~ happened, but suddenly, treacherous dust particles entered the traveler's spiracles, and he sneezed loudly, accidentally hitting some lever on the bathyscaphe's control panel. As a result, the bathyscaphe began moving vertically upward with an acceleration a.

1. *(1 point)* Assuming the average density of the bathyscaphe (together with Little Ant) is less than the density of the liquid, find the value of a at which the aquarium will completely lift off ~~for good~~ from the floor.

The noise was heard from the other side of the door, and the knocking grew even louder.

"You're flooding us!"—our traveler heard the voice of the downstairs neighbor.

Part 4. Ethics and Insectism

After the sudden vertical launch and the collision with the aquarium lid, another hidden compartment of the bathyscaphe opened, and ~~nearly getting knocked out~~ a hefty tome titled 'Ethics and Insectism in Experiments on Insects' tumbled out. Hitting the floor of the bathyscaphe, the book fell open to a random page. As Little Ant regained his composure, he began reading with growing interest, and what he read made his antennae stand on end.

At the bottom of a small, sealed test tube suspended by a thread above a table sits a Dragonfly, whose mass is twice that of the test tube. The distance from the bottom of the tube to the table's surface equals the length of the tube, l. The thread is swiftly cut. During the fall, the Dragonfly flies from the bottom to the very top of the test tube.

1. *(1 point)* Determine the time after which the bottom of the test tube will hit the table.

Horrified, Little Ant slammed the tome shut, shut down the bathyscaphe's engines (after docking it back in its standard starting position near the center of the left wall), put on a diving suit, and—remembering that he was, in fact, a terrestrial creature—climbed out of the fully enclosed vessel.

~~Exhausted from solving the main problem~~ Overwhelmed by stress, Little Ant decided to meditate. He sat on the edge of the aquarium and began methodically dropping pebbles onto the floor.

Part 5. Flight Debriefing

Little Ant is sitting at a height of $h_0 = 1$ m and is throwing stones at different angles α with speed v.

1. *(1 point)* Find the times t_1, t_2, t_3 corresponding to the stone hitting the floor, reaching the height $2h_0$, and moving a horizontal distance S.

2. *(1 point)* Analyze which of these events happens first (depending on the angle α), if $v = 5$ m/s, $S = 3$ m, $h_0 = 1$ m. Assume the acceleration due to gravity is $g = 10$ m/s^2.

Part 6. Friction with impact

A body[2] of mass m approaches a horizontal surface with speed v_0 directed at an angle α to the vertical. The coefficient of friction between the surface and the body is μ. Assuming that after the collision the vertical component of the body's momentum becomes zero, find:

1. *(1 point)* assuming the angle $\alpha > \arctan \mu$, find the change in horizontal momentum during the collision.

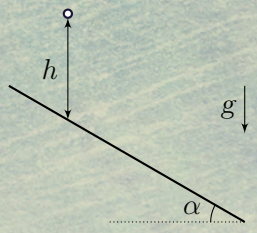
Let the horizontal surface be a board of mass M (with $M > m$), which is resting on a smooth horizontal surface.

2. *(2 points)* Find the change in the box's velocity during the collision, if the angle $\alpha > \arctan(2\mu)$.

Wishing to see the situation from a different perspective, Little Ant climbed onto a moored bathyscaphe with a batch of pebbles taken from the bottom of the aquarium and started having fun.

Part 2π. Déjà vu

The downstairs neighbor played the accordion beautifully. A material point falls onto an inclined plane from height h with no initial velocity. The collisions between the point and the plane are perfectly elastic. The friction coefficient between the point and the plane is μ. Air resistance should be neglected.



1. During the motion time the point reaches the height of the first collision three times (taking into account the first one). Find the angle α between an inclined plane and horizontal in the following cases

   а) *(0 points)* $\mu = 0$;

   б) *(0 points)* $\mu > \operatorname{tg} \alpha$.

2. Assume $\alpha = \pi/6$. Find the displacement of the material point during the time $t \gg \sqrt{\dfrac{h}{g}}$ in the following cases

   а) *(0 points)* $\mu = 0{,}5$;

   б) *(0 points)* $\mu = 0{,}8$.

[2] This body was Ant's stone.