



LPR Cup 2023

10.s04.e02

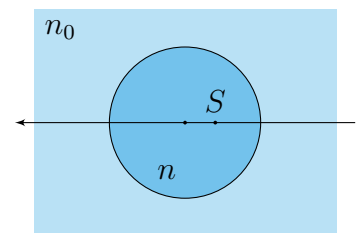
Look closely, because the closer you think you are, the less you will actually see.

Now You See Me (movie)

Distortion of Hidden

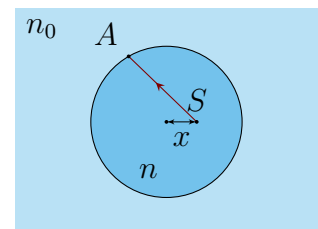
In a medium with a refractive index $n_0 = 1,20$, there is the Ball of radius R made of a material with a refractive index $n = 1,80$.

- (1 point) Hidden S is located at a distance $R_2 = R/2$ from the center of the Ball. Where is the image that Seeker sees if he is located to the left of the Ball on the optical axis of the system (see figure)? Give the answer in the paraxial approximation.
- (1 point) Seeker looks with wide-open eyes, so that he sees even those rays that came out from Hidden at an angle $\pi/3$ to the optical axis. At what distance from the center of the Ball will the continuations of these rays intersect the optical axis?



The continuations of the rays do not corradiate, therefore, Hidden looks Distorted, so Seeker understands that this is just an image and continues the Game. Further in the problem, we will have to find, where should Hidden be located inside the Ball, so that the continuations of all rays (both in the paraxial approximation and not) corradiate.

Consider the refraction of a ray emitted by Hidden from a point S when exiting the Ball at some point A . Let's denote the distance from Hidden to the center of the Ball as x , and the distance from the center of the Ball to the intersection point S' of a straight line, containing a refracted ray, and the optical axis as y .



- (4 points) Prove that $\frac{SA}{nx} = \frac{S'A}{n_0y}$.
- (3 points) Find the values of x at which the images of Hidden will not be Distorted, and he will outsmart Seeker.
- (1 point) Where will the image of Hidden be in this case?

Consider Hidden is a point-source, which radiates only into the left half-space, where Seeker is located. The border of the Ball does not reflect light. Trigonometric formulas can be used without proof.

First hint — 01.05.2023 20:00 (Moscow time)

Second hint — 03.05.2023 12:00 (Moscow time)

Final of the second round — 05.05.2023 20:00 (Moscow time)