



# LPR Cup 2023

10.s04.e01

## Hint 2

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0,7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where  $p_i$  is a point for the problem item, and  $k_i$  is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

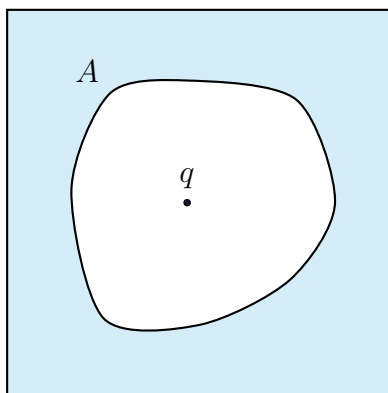
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

## Exercises

Tasks in this section will not be graded! Don't send their solutions! Please...

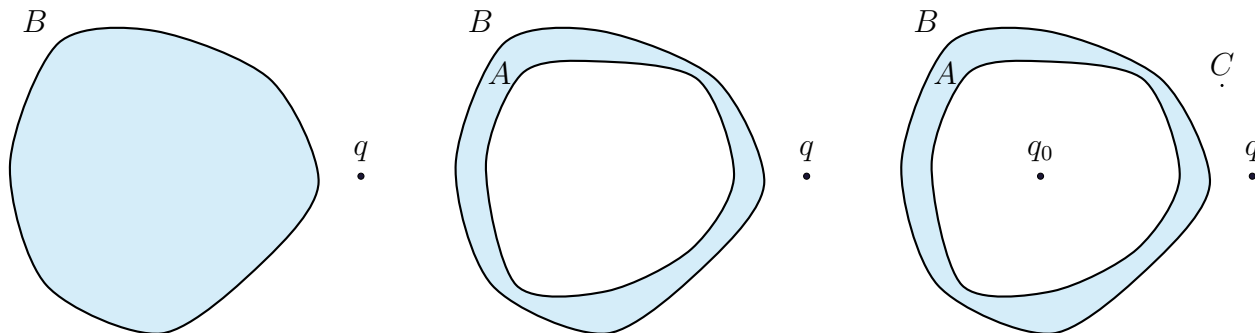
### General problems on conductors

**Exercise.** (0 points) A point charge  $q$  is placed inside the cavity of infinite conductor. The cavity has some shape  $A$ . What is the total charge of the cavity?



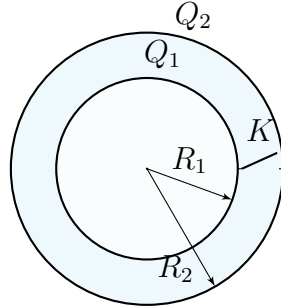
**NB.** For random shape of the conductor it is impossible to find the charge distribution, however, it uniquely exists. Assume that this distribution is given and further we call it distribution  $X$ .

**Exercise.** (0 points) The continuous conductor of shape  $B$  brings charge  $Q$  and is placed in the field of a point charge  $q$ . What will be the field inside the cavity of shape  $A$ , which is made in the conductor? How will the charge be distributed inside this cavity? What charge distribution will be inside this cavity, if a charge is placed there? What charge will be on the outer surface in this case? Will the charge distribution on the outer surface in this case be changed? Is it possible to find how the potential of the electric field at point  $C$ , created by the conductor, will change?



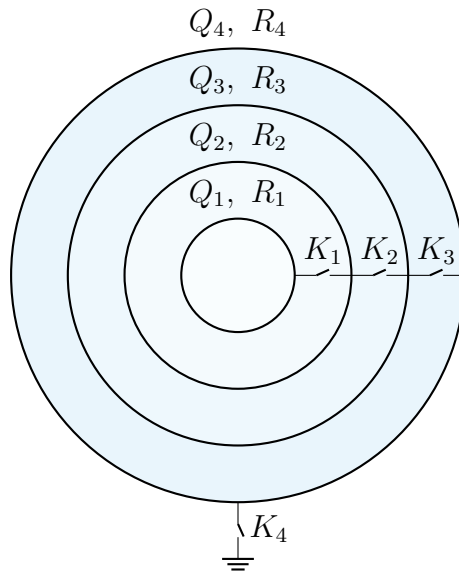
### Basic problems on spheres

**Exercise.** (0 points) Two concentric spheres with radii  $R_1$  and  $R_2$  bring charges  $Q_1$  and  $Q_2$  accordingly. What charge will flow in the circuit after the toggle switch is closed.



**Exercise.** (0 points) Four concentric spheres of radii  $R_1 < R_2 < R_3 < R_4$  are charged with charges  $Q_1, Q_2, Q_3$  and  $Q_4$  accordingly. All toggle switches are open. Assume that in all questions of this problem, before the certain toggle switchers are closed, the system is returned in the initial state, that is, all toggle switches are open, the spheres are charged as it was before one or the other toggle switcher was closed.

1. What charge will flow through the toggle switcher  $K_1$  after its closure?
2. What charge will flow through the toggle switcher  $K_2$  after its closure?
3. What charge will flow through the toggle switcher  $K_3$  after its closure?
4. What charge will flow through the toggle switcher  $K_4$  after its closure?



**Exercise.** (0 points) A cube with a side  $a$  is uniformly charged by its volume. The potential at its vertex equals  $\varphi$ . Find the potential at its vertex, if all its linear dimensions are halved, and the charge density remains the same? How will the answer change, if it is the charge which remains the same? How will the answer change, if the cube is charged by its surface? By its ribs?

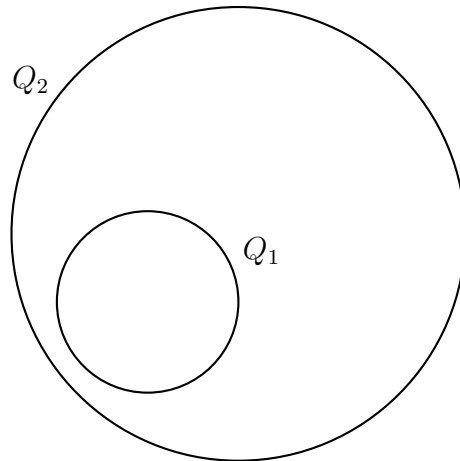
**Exercise.** (0 points) Find the potential in the centre of the uniformly charged cube from the previous problem.

## Alternative problem

You might want to send us solutions for the tasks which are rated at zero points. Or you might not. It's for you to decide.

**Task 1.** Two spheres are located as it shown in the picture. The charge of the inner sphere equals  $Q_1$ , the charge of the outer one is  $Q_2$ .

1. (2 points) What charge will flow in the wire, if the toggle switch gets closed?
2. (0 points) Will the answer change, if there is no space filled with conductor instead toggle switcher, which is placed between, gets closed?



**Task 2.** (3 points) The charges from the previous problem are oppositely charged and have the same absolute value  $Q$ . The difference of potentials between the spheres is  $\Delta\varphi$ . How much heat will be released on the wire after we connect them with it? Assume that the resistance of the wire is much greater than the resistance of the spheres

**Task 3.** Slabs 1 and 4 are connected to the voltage source. Zero potential is chosen so that the potential of the slab 1 equals  $\varphi_{\text{in}}$ , slab 4 —  $\varphi_{\text{out}}$ . Total charge of slabs 1 and 4 equals zero, charges of slabs 2 and 3 equal  $q_2$  and  $q_3$  accordingly. Area of slabs is  $S$ , distance between neighbouring slabs is  $d$ .

1. (1 point) Find the surface charge density on slab 2.
2. (1 point) Find the surface charge density on slab 3.

Let the part of the space left to slab 1 be called «inner». The part of the space right to slab 4 be called «outer».

3. (1.5 points) Find the field, created by the left surface of slab 2 in «inner» part of space.
4. (1.5 points) Find the field, created by the right surface of slab 2 in «outer» part of space.
5. (0 points) How is it connected with the charge distribution on the Cube? On Cuba? ~~Chilli?~~ ~~Chilling?~~

Edge effects can be neglected.

